MATHEMATICS Preparation 10

Number Connections

Module 2

Learning Technologies Branch

Alberta LEARNING

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Mathematics Preparation 10
Module 2: Number Connections
Student Module Booklet
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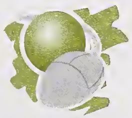
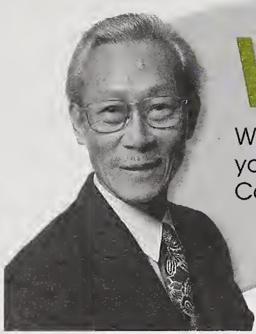
This document is intended for	
Students	✓
Teachers	✓
Administrators	
Parents	
General Public	
Other	



The Learning Technologies Branch has an Internet site that you may find useful. The address is as follows:

http://www.learning.gov.ab.ca/ltb

The use of the Internet is optional. Exploring the electronic information superhighway can be educational and entertaining. However, be aware that these computer networks are not censored. Students may unintentionally or purposely find articles on the Internet that may be offensive or inappropriate. As well, the sources of information are not always cited and the content may not be accurate. Therefore, students may wish to confirm facts with a second source.



Welcome

Welcome to Module 2. We hope you enjoy your study of "Number Connections."

Course Overview

Mathematics Preparation 10 consists of five modules and a final test. Each module is worth 1 credit. Discuss with your teacher how many modules you need to complete to prepare yourself for Pure Mathematics 10 or Applied Mathematics 10.

Mathematics Preparation 10

Module 1
Exploration of Numbers

Module 2
Number Connections

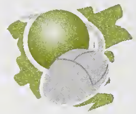
Module 3
Patterns and Algebra

Module 4
Polynomials

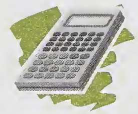
Module 5
Shape and Space

The document you are now reading is called a Student Module Booklet. It has accompanying Assignment Booklets.

You will find many visual cues or icons throughout this Student Module Booklet. Read the following statements to discover what each icon prompts you to do.

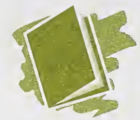


Use the Internet to explore a topic.



Use a scientific calculator.

Use the suggested answers in the Appendix to correct activities.



Answer the questions in the Assignment Booklet.

Throughout the course, you will be given instruction and practice on each of the following mathematical processes:

- connecting mathematical ideas
- developing and using estimation strategies
- developing and using mental math strategies
- using technology appropriately
- developing and using problem-solving strategies
- reasoning and justifying your answers
- using visualization to assist in processing information
- solving problems
- communicating mathematically

For example, to help you develop your problem-solving skills, several problem-solving strategies are explained in the Appendix of each Student Module Booklet, and you will be given several non-routine problems to solve in each module. Watch for the heading "Now Try This."

To help you develop your communication skills in mathematics, you will be asked to do several journal writings in each module. Watch for the heading "Looking Back."

Contents

Module 2: Number Connections

Module Overview

Assessment 1
Strategies for Completing the Module Successfully 2

Section 1: Fractions and Mixed Numbers

Activity 1: Developing a Number Sense for Fractions 4
Activity 2: Adding and Subtracting Fractions and Mixed Numbers 24
Activity 3: Multiplying Fractions and Mixed Numbers 37
Activity 4: Dividing Fractions and Mixed Numbers 47
Activity 5: Using a Calculator for Operations on Fractions and Mixed Numbers 59
Conclusion 67
Assignment 67

Section 2: Using Fractions: Rates, Ratios, and Proportions

Activity 1: Ratios 69
Activity 2: Rates 75
Activity 3: Percents 83
Activity 4: Solving Ratio, Rate, and Percent Problems 95
Conclusion 102
Assignment 102

Section 3: Data, Bivariate Data, and Scatter Plots

Activity 1: What Is Statistics? 104
Activity 2: Examining Bivariate Data 109
Activity 3: Making a Scatter Plot 112
Activity 4: Drawing Lines of Best Fit and Analysing Scatter Plots .. 119
Conclusion 126
Assignment 126

Module Summary 126

Appendix

Problem-Solving Strategies 128
Glossary 149
Suggested Answers 151
Credits 200

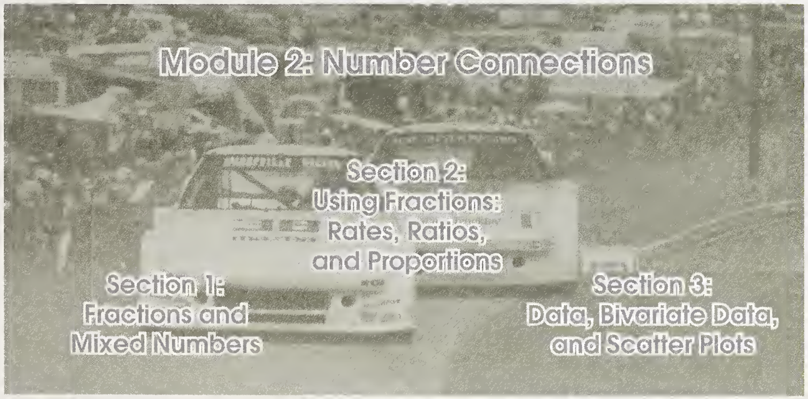
NOVA DEVELOPMENT CORPORATION

Module Overview

Do you follow auto racing? If you do, you certainly have heard of Canada's Jacques Villeneuve. Born in 1971 in St. Jean-Sur-Richelieu, Quebec, he began his formal racing training at the age of 15. By 1996, he was the Formula One runner up; and, in 1997, he was the Formula One world champion.

The world of auto racing is one of numbers. Lap times are quoted in hundredths of a second and speeds in thousandths of a kilometre per hour. Did you know that in Formula One racing, for example, in order to qualify for a race, your lap time in the practice round must be no more than 1.07 times than that posted by the fastest qualifier? The maximum and minimum dimensions of the race car, its engine capacity, and its weight are only a few of the other guidelines that race officials strictly enforce.

The world of numbers and statistics are also the topics of this module. In Section 1, you will review fractions and their operations; in Section 2, you will use fractions to explore rates, ratios, and proportions; and in Section 3, you will analyse data.

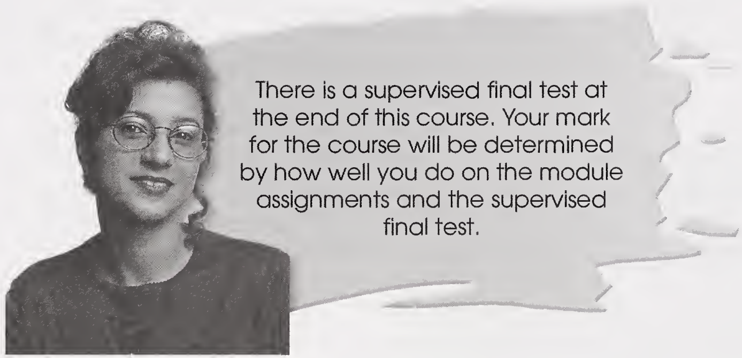


Assessment

Your mark for this module will be determined by how well you complete the assignments at the end of each section. In this module, you must complete three assignments. The mark distribution is as follows:

Assignment Booklet 2A	
Section 1 Assignment	**39 marks**
Assignment Booklet 2B	
Section 2 Assignment	**30 marks**
Assignment Booklet 2C	
Section 3 Assignment	**31 marks**
TOTAL	**100 marks**

When doing the assignments, work slowly and carefully. You must do each assignment independently; but if you are having difficulties, you may review the appropriate section in this Student Module Booklet.



Stragegies for Completing the Module Successfully

To achieve success in this module, be sure to work slowly and systematically through the Student Module Booklet. Remember, your work in the Student Module Booklet will prepare you for your module assignments and final test.

Following are some strategies for completing the module successfully:

- Try to set realistic goals for each day; and once you've set these goals, stick to them.

- Read all the instructions in each Student Module Booklet carefully; answer all the questions in your mathematics binder; and check your answers by comparing your responses with the suggested answers in the Appendix.

- Keep a section of your mathematics binder for your journal entries. Get in the habit of describing new concepts, procedures, and strategies in your own words. Record useful ways to help you remember what a concept means. Make graphic organizers (such as context webs, Venn diagrams, and charts) to help you connect mathematical ideas.

- Ask someone who is taking Mathematics Preparation 10 to be your study partner. You will find that having a friend to discuss mathematics with will make your studying more enjoyable.

- If you need assistance from your teacher or your study partner, you may find it helpful to write your questions in the journal section of your mathematics binder before speaking to your teacher or study partner. This will help you pinpoint your problem.

- Take care when completing your module assignments. Be sure you have completed each part of the Assignment Booklets, and proofread the assignments before you submit them. Remember to review your module assignments after they are corrected; then file them in your binder so you can review your assignments and mathematics notes before you write the final test.

Good luck!

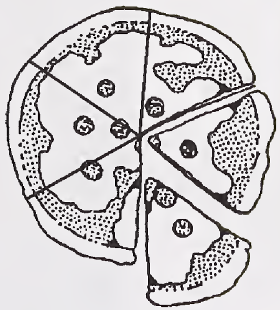
SECTION 1
Fractions and Mixed Numbers

Do you enjoy taking photographs? Have you ever used a 35-mm camera? Some 35-mm cameras have automatic settings; however, many photographers prefer to use cameras in which they select options, such as shutter speed.

The shutter speed controls the amount of light allowed to enter the camera. The numbers 2, 4, 8, 15, 30, 60, 125, 250, and 500 on a camera dial are shutter speeds and represent the fractions $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{15}, \ldots, \frac{1}{500}$ (of a second). For example, if a photographer sets the opening at 60, the shutter stays open for $\frac{1}{60}$ s.

In this section, you will study fractions. An understanding of fractions is vital to your understanding of mathematics. You will perform operations on fractions and mixed numbers and use models and pictures to help visualize the operations. Although today's calculators do perform operations on fractions, you should do the questions without a calculator unless specified. All of the examples and questions use operations on numbers that can easily be done with paper and pencil.

Activity 1: Developing a Number Sense for Fractions



You have been working with fractions for several years now. Fractions are important because not all quantities can be expressed by whole numbers. For example, six people want to share a pizza so that each person gets an equal portion. Each person will get one-sixth of the pizza.

The fraction one-sixth means "$1 \div 6$" or "1 of 6 equal parts"; it is written as $\frac{1}{6}$.

The top number in a fraction shows the number of parts being referred to, and the bottom number shows the total number of equal parts in the whole.

The top number of a fraction is the dividend, or more commonly called the numerator. The bottom number of a fraction is the divisor, or more commonly called the denominator.

$$\text{fraction}\left\{\frac{1}{6}\right. \begin{array}{l} \leftarrow \text{numerator} \\ \leftarrow \text{denominator} \end{array}$$

Example

What fraction of this set of cans is labelled as orange juice?

Solution

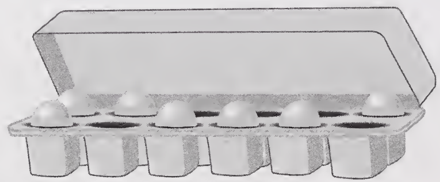
The number of cans labelled as orange juice is 2.

There are 3 cans altogether.

So, $\frac{2}{3}$ of the cans are labelled as orange juice.

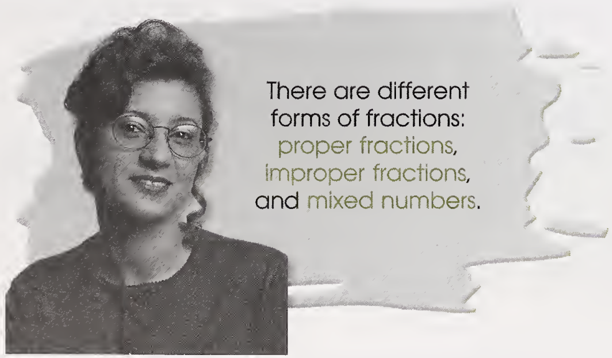
The fraction $\frac{2}{3}$ is read as "two-thirds."

1. a. There are 12 eggs in a full carton of eggs. Write a fraction to indicate the part of this carton that is filled.



b. There are 24 cans in a full case. Write a fraction to show the part of this case that is filled.

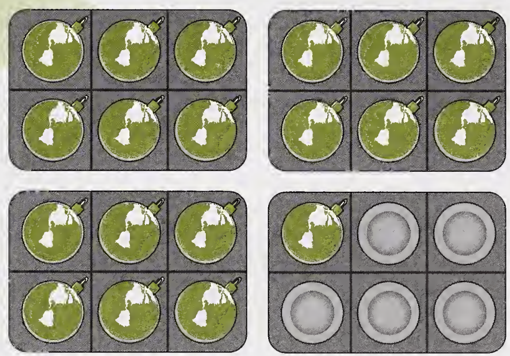
Check your answers by turning to the Appendix, page 151.



A proper fraction is a fraction in which the numerator is less than the denominator. An improper fraction is a fraction in which the numerator is greater than the denominator. A mixed number is a number expressed as a sum of a whole number and a proper fraction.

Example

What fraction of the boxes contain ornaments? Express your answer as an improper fraction and as a mixed number.



Solution

Find the improper fraction.

The number of sections filled with ornaments is 19.

The number of sections in each box is 6.

So, $\frac{19}{6}$ boxes contain ornaments.

Now, find the mixed number.

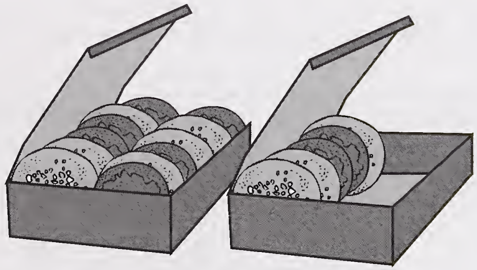
The number of boxes completely filled with ornaments is 3.

There is $\frac{1}{6}$ of another box filled with ornaments.

$$\therefore\ 3+\frac{1}{6}=3\frac{1}{6}$$

Therefore, $3\frac{1}{6}$ boxes contain ornaments.

2. What fraction of the cartons contain doughnuts? Express your answer as an improper fraction and as a mixed number.

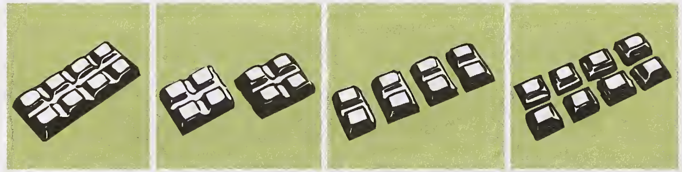


Check your answers by turning to the Appendix, page 151.

Equivalent Fractions

A person often answers to a variety of different names. For example, a person by the name of Betty may be called "Mom" or "Mother" by her children and "Aunt" or "Auntie" by her nieces and nephews.

Numbers also have many different names. For example, these fractions are all names of a whole chocolate bar.



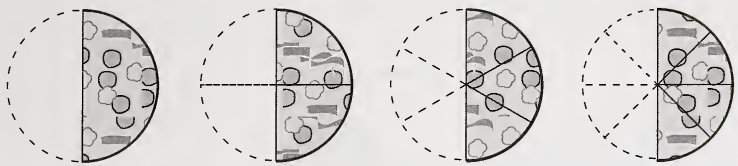
1 $\qquad \frac{2}{2} \qquad \frac{4}{4} \qquad \frac{8}{8}$

From the diagram you can see that there are many equivalent names for 1:

$$1=\frac{2}{2}=\frac{4}{4}=\frac{8}{8}$$

When fractions name the same part of a whole, they are said to be equivalent fractions.

Here is another example of equivalent fractions. These fractions name the same part of a whole pizza.



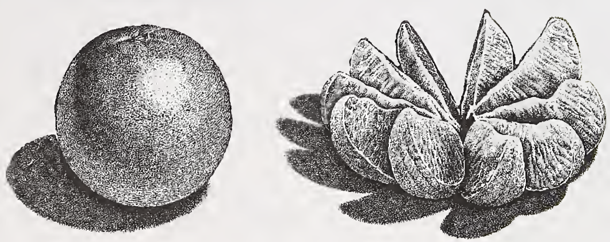
$\frac{1}{2}$ $\frac{2}{4}$ $\frac{3}{6}$ $\frac{4}{8}$

From the diagram, you can see that there are many equivalent names for $\frac{1}{2}$:

$$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8}$$

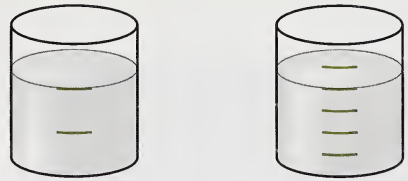
3. Write the equivalent names for the parts shown in each diagram.

a.



$1 = \frac{\square}{\square}$

b.



$\frac{2}{3} = \frac{\square}{\square}$

Check your answers by turning to the Appendix, page 151.

Example

Write three equivalent fractions for $\frac{6}{12}$.

Solution

Write equivalent fractions by dividing the numerator and the denominator by the same number.

The numerator 6 and the denominator 12 are each divisible by 2, 3, and 6.

$$\frac{6 \div 2}{12 \div 2} = \frac{3}{6} \qquad \frac{6 \div 3}{12 \div 3} = \frac{2}{4} \qquad \frac{6 \div 6}{12 \div 6} = \frac{1}{2}$$

$\therefore \frac{6}{12} = \frac{3}{6} = \frac{2}{4} = \frac{1}{2}$

You can also find equivalent fractions by multiplying the numerator and the denominator by the same number.

$$\frac{6}{12} \overset{\times 2}{\underset{\times 2}{=}} \frac{12}{24} \qquad \frac{6}{12} \overset{\times 3}{\underset{\times 3}{=}} \frac{18}{36} \qquad \frac{6}{12} \overset{\times 4}{\underset{\times 4}{=}} \frac{24}{48}$$

$\therefore \frac{6}{12} = \frac{12}{24} = \frac{18}{36} = \frac{24}{48}$

You should see from the preceding example that a fraction can be represented by an unlimited number of equivalent fractions.

4. Give four equivalent fractions to describe the part of the clock that is shaded.

Check your answer by turning to the Appendix, page 151.

Basic Fractions

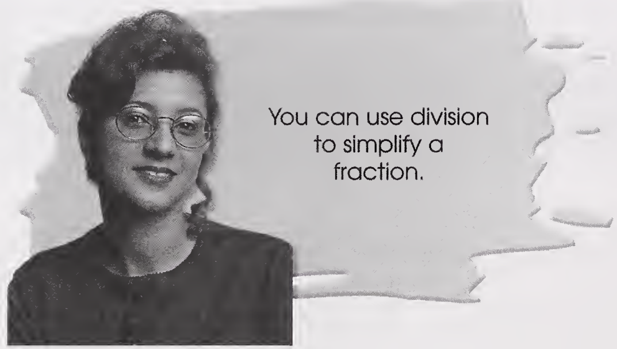
A fraction is in simplest form when it is written with the smallest possible whole-number denominator. A fraction in simplest form is also called a basic fraction.

The following are examples of basic fractions, thus they are in simplest form.

$\frac{1}{3}, \frac{2}{5}, \frac{7}{8}, \frac{9}{10}$

The following are examples of fractions that are not basic, thus they are not in their simplest form.

$\frac{4}{12}, \frac{6}{15}, \frac{14}{16}, \frac{45}{60}$



Example

Determine the basic fraction for $\frac{8}{12}$.

Solution

The basic fraction can be found by dividing the numerator and the denominator by the greatest common factor (GCF) of the two numbers.

$$\frac{8}{12} \overset{\div 4}{\underset{\div 4}{=}} \frac{2}{3}$$

The GCF of 12 and 8 is 4.

The smallest possible whole-number numerator of $\frac{8}{12}$ is 2. The smallest whole-number denominator of $\frac{8}{12}$ is 3. So, the simplest form of $\frac{8}{12}$ is $\frac{2}{3}$.

Therefore, $\frac{2}{3}$ is the basic fraction.

The simplest form of a fraction usually gives the clearest idea of the fraction's value.

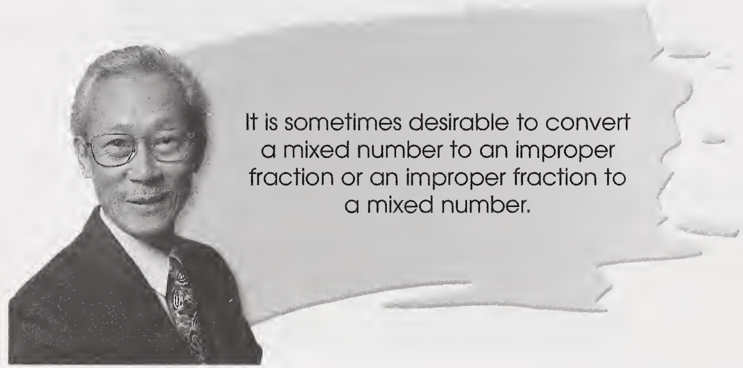
5. Write each of the following fractions in simplest form.

   a. $\frac{9}{12}$ b. $\frac{4}{6}$ c. $\frac{24}{96}$

   d. $\frac{72}{30}$ e. $\frac{30}{24}$ f. $\frac{45}{40}$

Check your answers by turning to the Appendix, page 151.

Converting Between Mixed Numbers and Improper Fractions



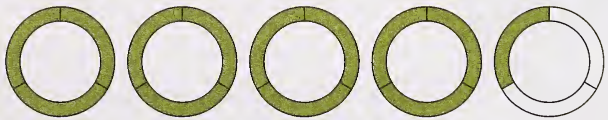
Example

Seth ran $4\frac{1}{3}$ laps around the track. Express $4\frac{1}{3}$ as an improper fraction.

Solution

Method 1

Draw four complete racetracks. Draw a fifth racetrack and mark or colour one-third of it. Since the fraction you are dealing with contains thirds, divide each of the four whole racetracks into thirds.



Count up the number of thirds in the $4\frac{1}{3}$ racetrack laps. Therefore, Seth ran $\frac{13}{3}$ laps around the track.

Method 2

$$4\frac{1}{3} = 4 + \frac{1}{3}$$
$$= 1+1+1+1+\frac{1}{3}$$
$$= \frac{3}{3}+\frac{3}{3}+\frac{3}{3}+\frac{3}{3}+\frac{1}{3}$$
$$= \frac{13}{3}$$

Seth ran $\frac{13}{3}$ laps around the track.

Method 3

Here is a quick way to find the improper fraction:

- Multiply the denominator of the fraction by the whole number. Then add the numerator of the fraction part to this product.
- Write the resulting number over the denominator of the original fraction.

whole number; denominator of fraction; numerator of fraction; denominator of fraction

$$4\frac{1}{3} = \frac{4\times 3+1}{3}$$
$$= \frac{12+1}{3}$$
$$= \frac{13}{3}$$

These steps can be done mentally.

Seth ran $\frac{13}{3}$ laps around the track.

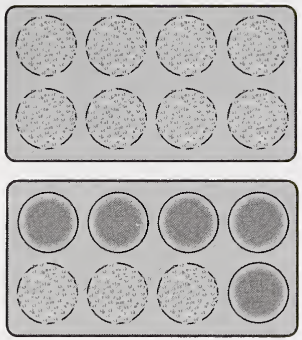
6. Express each of the following mixed numbers as an improper fraction.

a. $5\frac{1}{4}$ b. $3\frac{2}{5}$ c. $2\frac{1}{3}$ d. $4\frac{3}{5}$

Check your answers by turning to the Appendix, page 151.

Example

There are $\frac{11}{8}$ tins of muffins. Express $\frac{11}{8}$ as a mixed number.



Solution

Method 1

$$\frac{11}{8} = \frac{8}{8} + \frac{3}{8}$$
$$= \frac{8}{8} + \frac{3}{8}$$
$$= 1 + \frac{3}{8}$$
$$= 1\frac{3}{8}$$

11 eighths = 8 eighths + 3 eighths

There are $1\frac{3}{8}$ tins of muffins.

Method 2

Here is a quick way to find the mixed number:

- Divide the numerator of the improper fraction by the denominator.
- Write the whole number part of the quotient. Then divide the remainder by the denominator and write this part of the quotient as a fraction.

$$\begin{array}{r} 1 \\ 8\overline{)11} \\ 8 \\ \hline 3 \end{array}$$

denominator → 8; 11 ← numerator; 1 ← whole number part of quotient; 3 ← remainder

The remainder is divided by the denominator, too.

$$\therefore \frac{11}{8} = \text{whole number part of quotient} + \frac{\text{remainder}}{\text{denominator}}$$
$$= 1 + \frac{3}{8}$$
$$= 1\frac{3}{8}$$

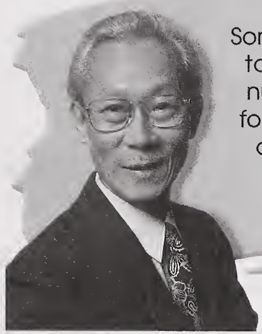
There are $1\frac{3}{8}$ tins of muffins.

7. Express each of the following improper fractions as a mixed number.

a. $\frac{10}{3}$ b. $\frac{16}{5}$ c. $\frac{21}{4}$ d. $\frac{33}{2}$

Check your answers by turning to the Appendix, page 151.

Converting Between Decimal Numbers and Fractions



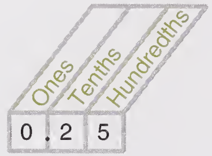
Sometimes it is desirable to change a decimal number to its fraction form or a fraction to its decimal form. Work through the next example.

Example

Fred took 0.25 h to cut out the pieces of material for his apron. Express 0.25 as a fraction in basic form.

Solution

Step 1: Use a place-value chart to help you make the conversion.



The decimal number 0.25 means 25 hundredths or $\frac{25}{100}$.

Step 2: Find the basic fraction by dividing the numerator and the denominator by their greatest common factor.

$$\frac{25 \div 25}{100 \div 25} = \frac{1}{4}$$

The GCF of 25 and 100 is 25.

Therefore, $0.25 = \frac{1}{4}$.

8. Express each of these decimal numbers as a fraction in simplest form.

a. 0.3	**b.** 0.26	**c.** 0.05
d. 4.25	**e.** 2.875	**f.** 3.036

Check your answers by turning to the Appendix, page 151.

You can use division to change a fraction to a decimal number.

Example

A store takes $\frac{1}{8}$ off the price of its merchandise. Express this fraction as a decimal number.

Solution

Step 1: Divide the numerator by the denominator. You may use long division or a calculator. Keep dividing until there is a remainder of zero or the remainders start to repeat.

$\frac{1}{8}$ can mean $1 \div 8$.

$$
\begin{array}{r}
0.125 \\
8\overline{)1.000} \\
\underline{8} \\
20 \\
\underline{16} \\
40 \\
\underline{40} \\
0
\end{array}
$$

← There is a remainder of zero.

1 ÷ 8 =

0.125

Step 2: Write the fraction as a decimal number.

$$\frac{1}{8} = 0.125$$

The store takes 0.125 off the price of its merchandise.

When you divide the numerator of a fraction by the denominator and there is a remainder of zero, the resulting decimal number is called a terminating decimal number.

9. Express each of these fractions as a decimal number. A calculator should not be necessary.

a. $\frac{3}{4}$	**b.** $\frac{2}{5}$	**c.** $1\frac{3}{10}$	**d.** $2\frac{1}{2}$

Check your answers by turning to the Appendix, page 151.

Example

Ryan helps coach his sister Lucy's baseball team. Lucy hit the baseball safely $\frac{5}{11}$ of the time. Express this fraction as a decimal number.

Solution

Step 1: Divide the numerator by the denominator. You may use long division or your calculator. Keep dividing until there is a remainder of zero or the remainders start to repeat.

$$
\begin{array}{r}
0.4545 \\
11\overline{)5.0000} \\
\underline{4\,4} \\
60 \\
\underline{55} \\
50 \\
\underline{44} \\
60 \\
\underline{55} \\
5
\end{array}
$$

← This remainder 6 occurred before.

← This remainder 5 occurred before.

5 ÷ 1 1 =

0.4545454

Step 2: Write the fraction as a decimal number.

$$\frac{5}{11} = 0.\overline{45}$$

The bar in the decimal $0.\overline{45}$ indicates that 45 will keep repeating indefinitely.

When you divide the numerator by the denominator and the remainders start to repeat, the resulting decimal number is a repeating decimal number.

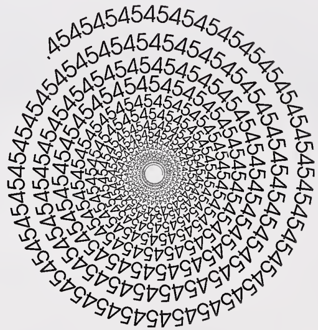
A repeating decimal number has an infinite number of non-zero digits with a repeating pattern.

A repeating decimal number can be written in either of the following ways. However, it is usually written with a bar placed over the repeating block of digits.

$$\frac{5}{11} = 0.\overline{45} \qquad \frac{5}{11} = 0.4545\ldots$$

The three dots (ellipsis) indicate the pattern continues.

The bar or dots are necessary. Regardless of the number of 45s you write or how you try to squeeze them in, at some time you will have to stop, thus making your result an approximation and not exactly equal to $\frac{5}{11}$!

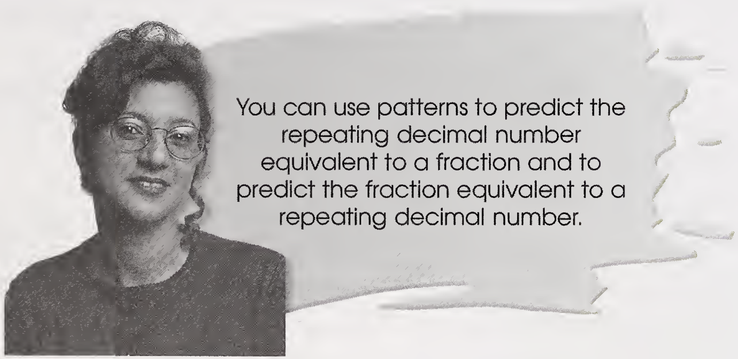


10. Write each of the following fractions as a decimal number. You may use a calculator. If the fraction is equivalent to a repeating decimal, use the bar notation to clearly show the repeating block of digits.

a. $\frac{7}{9}$ **b.** $\frac{5}{12}$ **c.** $\frac{9}{16}$

d. $\frac{9}{11}$ **e.** $\frac{3}{40}$ **f.** $\frac{2}{13}$

Check your answers by turning to the Appendix, page 151.



11. Answer questions 11.a. to 11.c. using the following series of fractions:

$$\frac{1}{9}, \frac{2}{9}, \frac{3}{9}, \frac{4}{9}$$

a. Convert each of the fractions in the series to a repeating decimal number. You may use a calculator.

b. What pattern do you see?

c. Use the pattern to predict a repeating decimal number equivalent to each fraction in this series:

$$\frac{5}{9}, \frac{6}{9}, \frac{7}{9}, \frac{8}{9}$$

12. Answer questions 12.a. to 12.c. using the following series:

$$\frac{1}{99}, \frac{2}{99}, \frac{3}{99}, \frac{4}{99}, \frac{5}{99}$$

a. Convert each of the fractions in the series to a repeating decimal number. You may use a calculator.

b. What pattern do you see?

c. Use the pattern to predict a repeating decimal number equivalent to each fraction in this series:

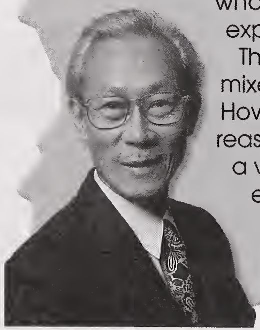
$$\frac{13}{99}, \frac{23}{99}, \frac{47}{99}, \frac{68}{99}, \frac{94}{99}$$

13. Use the patterns you discovered in questions 11 and 12 to find a fraction equivalent to each of the following repeating decimal numbers.

a. $2.\overline{7}$ **b.** $0.\overline{14}$ **c.** $3.\overline{26}$ **d.** $1.\overline{8}$

Check your answers by turning to the Appendix, page 152.

Problem Solving



Many problems involve dividing two whole numbers. The way the answer is expressed depends on the situation. The answer could be a fraction, a mixed number, or a decimal number. However, in some situations, it is more reasonable to round off the answer to a whole number. The answer could even be the remainder in some situations.

Example

Charlie's marching band rehearsed a total of 29 h in the last 4 days before the parade. If they spaced their rehearsals equally over the 4 days, how many hours did they practise each day?

Solution

Divide the total number of hours by the number of days.

$$\begin{array}{r} 7\text{R}1 = 7\tfrac{1}{4} \\ 4\overline{)29} \\ \underline{28} \\ 1 \end{array} \qquad \begin{array}{r} 7.25 \\ 4\overline{)29.00} \\ \underline{28} \\ 1\,0 \\ \underline{8} \\ 20 \\ \underline{20} \\ 0 \end{array}$$

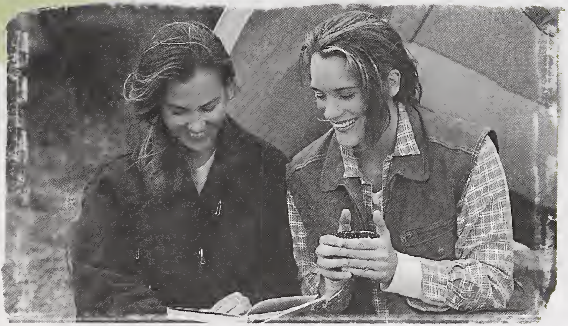
2 9 ÷ 4 =

7.25

The band practised $7\frac{1}{4}$ h or 7.25 h each day.

Example

A class of 29 students at Willow Creek School are going on a field trip. If 4 students fit in each car, how many cars are needed?



Solution

$$4\overline{)29} = 7\text{R}1 = 7\tfrac{1}{4}$$
$$28$$
$$1$$

$$4\overline{)29.00} = 7.25$$

28
10
8
20
20
0

2 9 ÷ 4 =

7.25

It is not reasonable to have $7\frac{1}{4}$ cars or 7.25 cars; you can only have whole cars. The answer must be rounded up to 8 cars because 7 cars would be too few.

Therefore, the students will need 8 cars.

Example

Giulio has collected 29 cassette tapes. He wants to arrange them in a box that will hold 4 tapes in each row. How many tapes will he put in the last row?

Solution

Find the remainder.

$$4\overline{)29} = 7$$
$$28$$
$$1$$

← This is the remainder.

If you use a calculator, the answer will be displayed as a decimal number. You can find the remainder by subtracting the whole number part and then multiplying the decimal number part by the divisor.

2 9 ÷ 4 =

7.25

Divide.

− 7 =

0.25

Subtract the whole number part.

× 4 =

1.

Multiply by the divisor. This is the remainder.

There will be 1 tape in the last row.

14. A theatre group sold 36 tickets for a total of $450. What was the price of one ticket?

15. The Egg Marketing Board receives a shipment of 4000 eggs. How many dozen eggs do they receive?

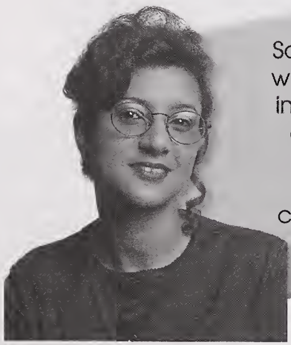
16. The teen club had a bottle drive to raise money. They collected 1335 soda cans. They put the soda cans in boxes where 24 cans fit in each box.

   a. How many boxes were needed?

   b. How many boxes were full?

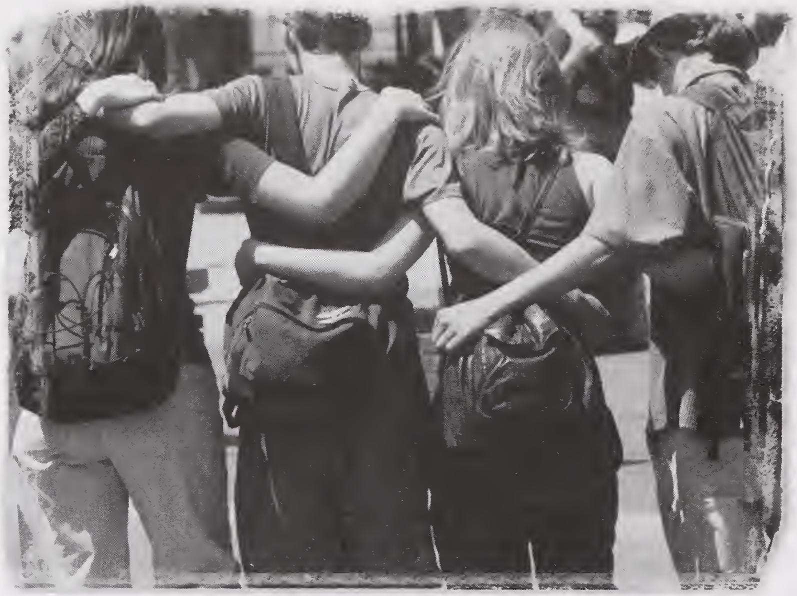
   c. How many bottles were in the partly filled box?

Check your answers by turning to the Appendix, page 152.



So far in this activity, you have written fractions and decimals in equivalent forms. You have converted between mixed numbers and improper fractions, and you have converted between decimal numbers and fractions.

Comparing and Ordering Fractions and Decimals



These friends all have different heights. If you lined up the friends in order of height from shortest to tallest, who would be first? Who would be last?

How do you compare and order fractions and decimal numbers?

You use the same symbols that you use for whole numbers.

$=$ (is equal to) $<$ (is less than) $>$ (is greater than)

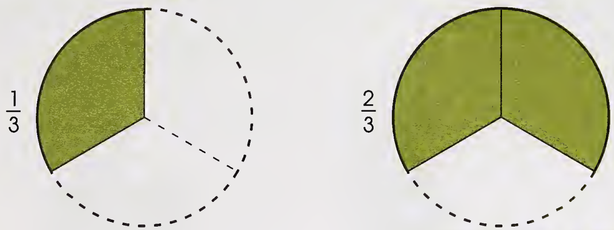
Begin by comparing fractions with like denominators.

Example

Dinh has $\frac{1}{3}$ of a pie, and Marshall has $\frac{2}{3}$ of a pie. Who has less pie?

Solution

Draw a diagram to illustrate the situation.



From the diagram, you can see that $\frac{1}{3}$ is less than $\frac{2}{3}$.

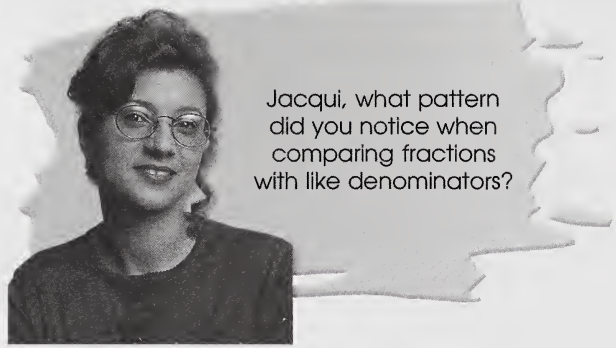
$\frac{1}{3} < \frac{2}{3}$ The inequality sign points to the number that is less.

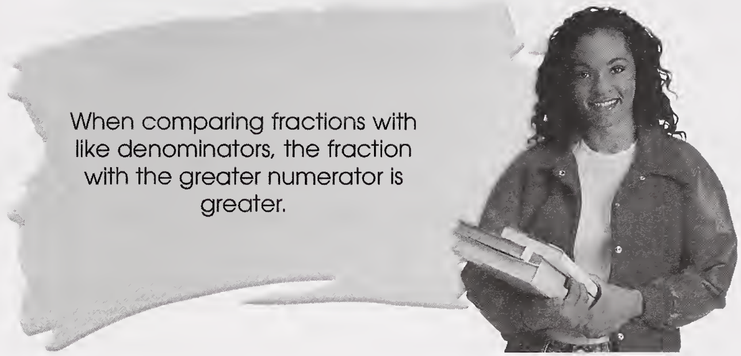
Dinh has less pie.

17. Use < or > to compare each pair of fractions.

a. $\frac{1}{6}$ ● $\frac{5}{6}$ **b.** $\frac{3}{4}$ ● $\frac{1}{4}$ **c.** $\frac{5}{12}$ ● $\frac{7}{12}$

Check your answers by turning to the Appendix, page 153.





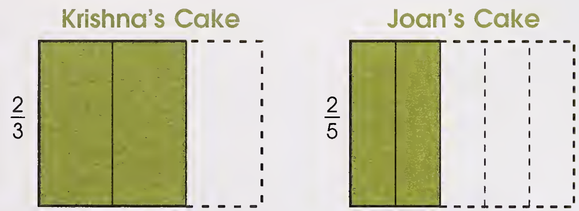
Now, you will compare fractions with like numerators.

Example

Krishna has $\frac{2}{3}$ of a cake, and Joan has $\frac{2}{5}$ of a cake. Who has more cake?

Solution

Draw a diagram to illustrate the situation.



From the diagram, you can see that $\frac{2}{3}$ is greater than $\frac{2}{5}$.

$\frac{2}{3} > \frac{2}{5}$ The inequality sign points to the number that is less.

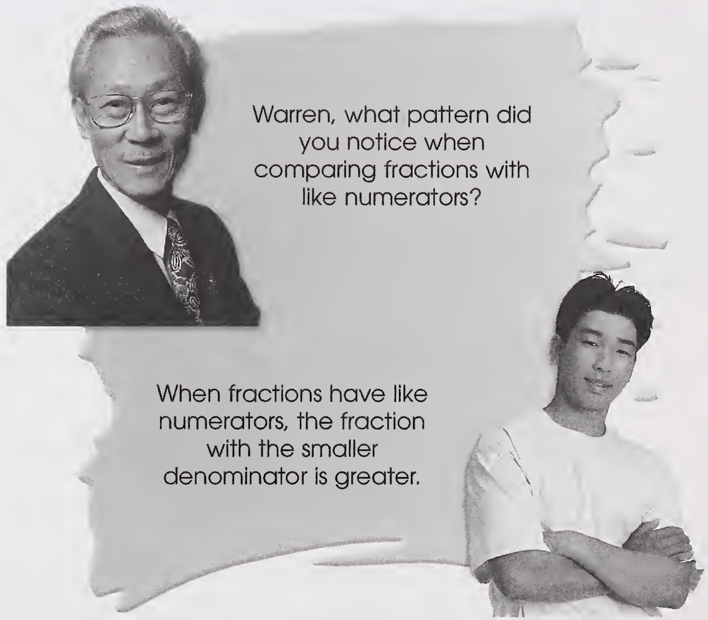
Krishna has more cake.

18. Use < or > to compare the fractions.

a. $\frac{1}{6}$ ⬤ $\frac{1}{2}$ **b.** $\frac{1}{2}$ ⬤ $\frac{1}{3}$

c. $\frac{5}{6}$ ⬤ $\frac{5}{12}$ **d.** $\frac{7}{4}$ ⬤ $\frac{7}{12}$

Check your answers by turning to the Appendix, page 153.

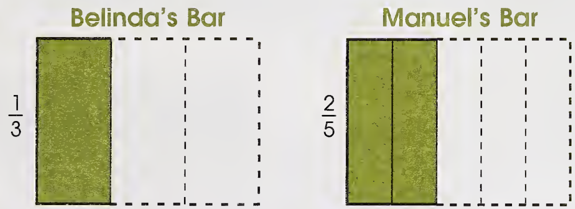


Example

Belinda has $\frac{1}{3}$ of a chocolate bar, and Manuel has $\frac{2}{5}$ of a chocolate bar. Who has more chocolate?

Solution

Draw a diagram to illustrate the situation.

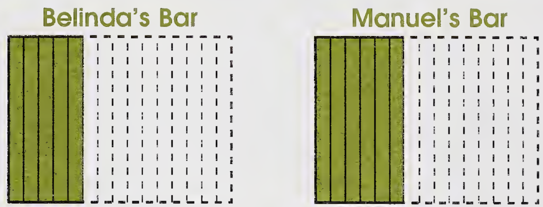


It is difficult to tell which person has more chocolate simply by looking at the diagram.

One way to make it easier to compare the amounts is to cut both chocolate bars into pieces that are the same size. By doing this, you would be renaming the fractions with common denominators.

You can find a common multiple of 3 and 5 by multiplying the two numbers. Thus, a common denominator will be 15.

$$\frac{1}{3} \overset{\times 5}{\underset{\times 5}{=}} \frac{5}{15} \qquad \frac{2}{5} \overset{\times 3}{\underset{\times 3}{=}} \frac{6}{15}$$



From the new diagram, you can see that $\frac{6}{15}$ is greater than $\frac{5}{15}$.

$$\frac{6}{15} > \frac{5}{15}$$

$$\therefore \frac{2}{5} > \frac{1}{3}$$

Manuel has more chocolate.

19. Use < or > to show which fraction is the greater of the two.

a. $\frac{2}{5} \bigcirc \frac{3}{7}$ **b.** $\frac{7}{9} \bigcirc \frac{2}{3}$

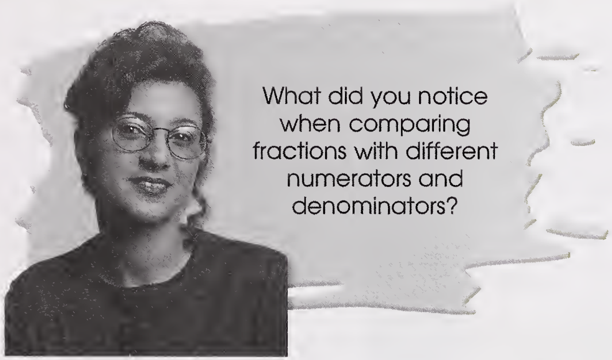
c. $\frac{3}{4} \bigcirc \frac{4}{5}$ **d.** $\frac{5}{6} \bigcirc \frac{11}{12}$

20. If it took Marnee $\frac{1}{2}$ h to complete one activity of the math course and it took Darwin $\frac{3}{4}$ h to complete the same activity, which person took more time to complete the work?

h = hour

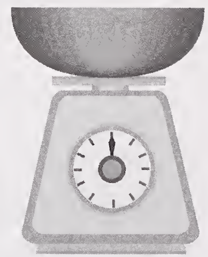
21. Tran, a diamond expert, bought a $\frac{1}{2}$-carat diamond, a $\frac{5}{8}$-carat diamond, and a $\frac{2}{3}$-carat diamond. Which is the biggest diamond of the three?

Check your answers by turning to the Appendix, page 154.



Example

Julian has one bag that contains 0.4 kg of soybeans and another that contains $\frac{1}{2}$ kg. Which bag weighs more?



Solution

Method 1: Finding Equivalent Fractions

Step 1: Find equivalent fractions with a common denominator.

$$0.4 = \frac{4}{10} \qquad \frac{1}{2} = \frac{5}{10}$$

Step 2: To decide which is greater, compare the fractions with common denominators.

$$\frac{5}{10} > \frac{4}{10}$$

$$\therefore \ \frac{1}{2} > 0.4$$

The $\frac{1}{2}$-kg bag weighs more.

Method 2: Finding Equivalent Decimal Numbers

Step 1: Find equivalent decimal numbers.

0.4 $\qquad$ $\frac{1}{2} = 0.5$

Step 2: To decide which is greater, compare the decimal numbers.

$0.5 > 0.4$

$\therefore \frac{1}{2} > 0.4$

The $\frac{1}{2}$-kg bag weighs more.

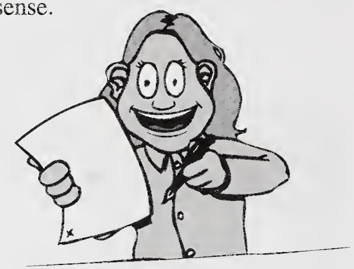
22. Use < or > to show which number is the greater of the two. You can change the fractions to decimals using a calculator.

a. $\frac{7}{8}$ ⬤ 0.75 $\qquad$ **b.** $\frac{5}{8}$ ⬤ 0.6

c. 0.6 ⬤ $\frac{2}{3}$ $\qquad$ **d.** $\frac{1}{7}$ ⬤ 0.2

Check your answers by turning to the Appendix, page 154.

Looking Back



In this activity, you developed your number sense. You wrote fractions to represent everyday situations to develop a meaning of fractions. You then wrote equivalent fractions, a necessary skill for adding and subtracting fractions. You represented fractions in many forms—as mixed numbers, as improper fractions, and as decimals and you converted between these forms. Finally, you ordered fractions and decimals.

23. In your journal, write a definition of the following terms in your own words and provide an example.

a. improper fraction

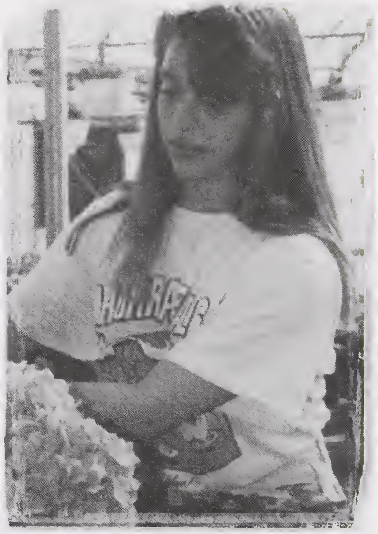
b. mixed number

24. In your everyday life, you may have heard someone say, "I'll get time-and-a-half if I work on the holiday" or "They scored the most points in the first quarter." Write these two statements in your journal; then explain the meaning of the numbers used in these statements.

Check your answers by turning to the Appendix, page 155.

Activity 2: Adding and Subtracting Fractions and Mixed Numbers

Francesca enjoys her part-time job at the greenhouse. On Monday, she worked $1\frac{1}{2}$ h. On Tuesday, she worked $2\frac{3}{4}$ h. Altogether, how much time did Franscesca work on these two days? How much longer did she work on Tuesday than on Monday?



By the time you finish this activity, you will be able to solve problems like this.

Many situations require the addition of fractions. There are different ways you can do these additions.

Example

Jack and Jill shared a pizza. Jack ate $\frac{3}{8}$ of the pizza and Jill ate $\frac{1}{8}$ of the pizza. What fraction of the entire pizza was eaten by Jack and Jill together?

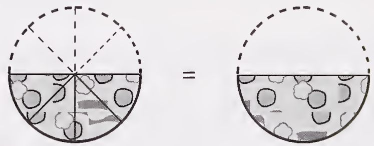
Solution

Method 1: Using a Diagram

Draw a diagram that shows Jack's share of the pizza. He ate 3 eighths.

Now, draw a diagram that shows Jill's share of the pizza. She ate 1 eighth.

Finally, draw a diagram that shows the fraction of the pizza eaten by Jack and Jill together.



3 eighths + 1 eighth = 4 eighths
= 1 half

Together, Jack and Jill ate $\frac{1}{2}$ of a pizza.

Method 2: Using Paper and Pencil

$$\frac{3}{8}+\frac{1}{8}=\frac{3+1}{8} \quad \longleftarrow \text{Add the numerators, and keep the common denominator.}$$
$$=\frac{4}{8}$$
$$=\frac{1}{2} \quad \longleftarrow \text{Simplify.}$$

Together, Jack and Jill ate $\frac{1}{2}$ of the entire pizza.

Problems that require the subtraction of fractions can also be solved in different ways.

Example

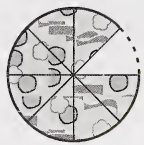
Ahmed and Jacques each ordered a pizza.

Ahmed ate $\frac{7}{8}$ of his pizza. Jacques ate $\frac{5}{8}$ of his pizza. How much more did Ahmed eat than Jacques?

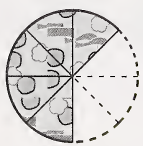
Solution

Method 1: Using a Diagram

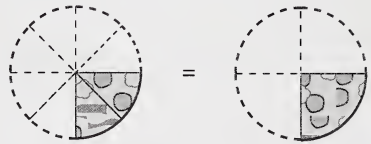
Draw a diagram that shows the fraction of Ahmed's pizza that was eaten. Ahmed ate 7 eighths of his pizza.



Now, draw a diagram that shows the fraction of Jacques's pizza that was eaten. Jacques ate 5 eighths of his pizza.



To determine how much more Ahmed ate than Jacques, subtraction is required.



7 eighths – 5 eighth = 2 eighths
= 1 fourth

Ahmed ate $\frac{1}{4}$ of a pizza more than Jacques.

Method 2: Using Paper and Pencil

$$\frac{7}{8}-\frac{5}{8}=\frac{7-5}{8} \quad \longleftarrow \text{Subtract the numerators, and keep the common denominator.}$$

$$=\frac{2}{8}$$

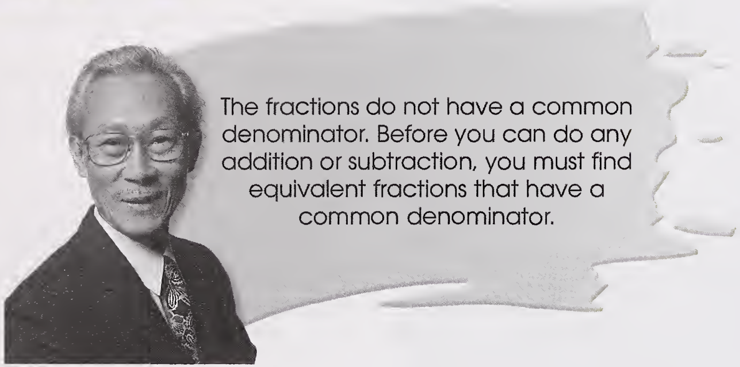
$$=\frac{1}{4} \quad \longleftarrow \text{Simplify.}$$

Ahmed ate $\frac{1}{4}$ of a pizza more than Jacques.

Example

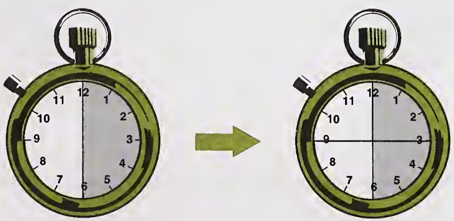
Rachel likes to write stories. On Tuesday, she wrote for $\frac{1}{2}$ h. On Thursday, she wrote for $\frac{3}{4}$ h. Altogether, how much time did Rachel spend writing?

Solution

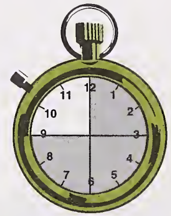


Method 1: Using a Diagram

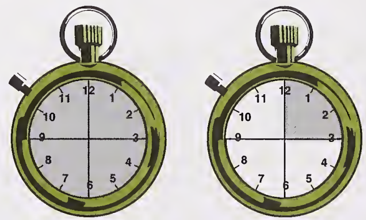
This diagram shows the time Rachel spent writing on Tuesday. Rachel wrote for $\frac{1}{2}$ h, or $\frac{2}{4}$ h.



This diagram shows the time Rachel spent writing on Thursday. She wrote for $\frac{3}{4}$ h.



The following diagram shows the total time Rachel spent writing.



2 fourths + 3 fourths = 5 fourths

Altogether, Rachel spent $\frac{5}{4}$ h or $1\frac{1}{4}$ h writing.

Method 2: Using Paper and Pencil

$$\begin{array}{r} \frac{1}{2} \\ +\ \frac{3}{4} \\ \hline \end{array} \quad \Rightarrow \quad \begin{array}{r} \frac{2}{4} \\ +\ \frac{3}{4} \\ \hline \frac{5}{4} = 1\frac{1}{4} \end{array}$$

Altogether, Rachel spent $1\frac{1}{4}$ h writing.

Example

Jenny and Zoe made lemonade. Jenny drank $\frac{3}{4}$ of a glass of lemonade, and Zoe drank $\frac{1}{2}$ of a glass. How much more lemonade did Jenny drink?

Solution

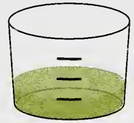
The fractions do not have a common denominator. You must find equivalent fractions that have a common denominator first.

Method 1: Using a Diagram

This diagram shows the amount Jenny drank. Jenny drank 3 fourths of a glass of lemonade.

This diagram shows the amount Zoe drank. Zoe drank 1 half, or 2 fourths, of a glass of lemonade.

The following diagram shows the difference.



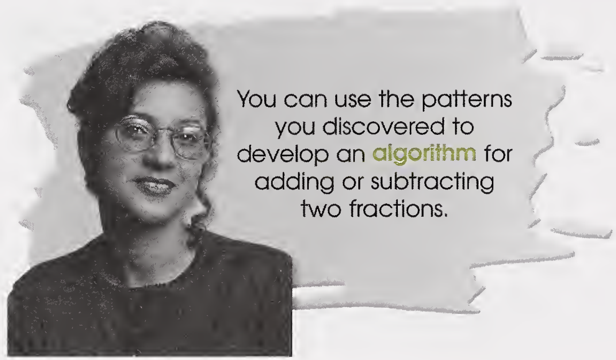
3 fourths – 2 fourths = 1 fourth

Jenny drank $\frac{1}{4}$ of a glass of lemonade more than Zoe.

Method 2: Using Paper and Pencil

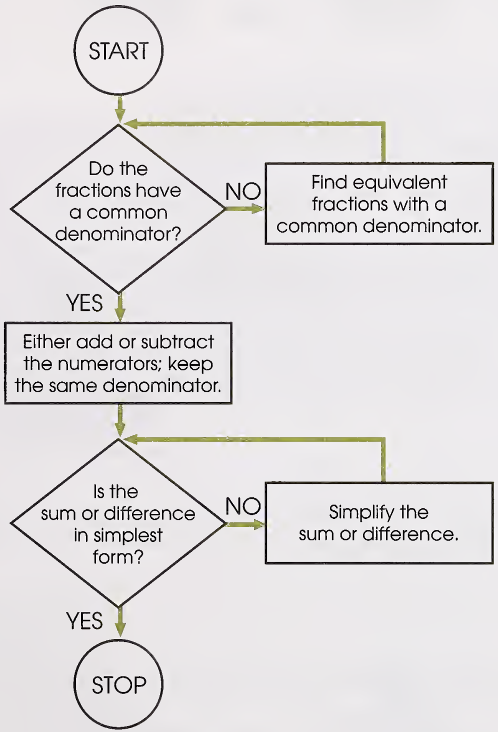
$$\begin{array}{r} \frac{3}{4} \\ -\ \frac{1}{2} \\ \hline \end{array} \Rightarrow \begin{array}{r} \frac{3}{4} \\ -\ \frac{2}{4} \\ \hline \frac{1}{4} \end{array}$$

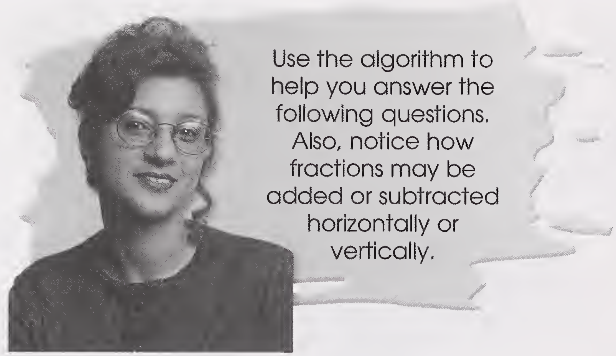
Jenny drank $\frac{1}{4}$ of a glass of lemonade more than Zoe.



An algorithm is a set of steps for finding the answer to a problem.

The algorithm for adding or subtracting two fractions is given in the following flow chart.





Use the paper-and-pencil method to answer questions 1 to 8.

1. On Friday, Mark spent $\frac{3}{8}$ of the day sleeping and $\frac{1}{4}$ of the day doing school work. What fraction of the day did Mark spend on these two activities?

2. Françoise had a jewellery collection that she wanted to share with her sisters. She decided to give $\frac{3}{10}$ of the collection to Aline and $\frac{5}{10}$ of the collection to Fernande. What fraction of her jewellery collection did Françoise give away?

3. Lucy and Ruth are forwards on a field hockey team. In the last game, Lucy scored $\frac{1}{2}$ of the goals and Ruth scored $\frac{1}{4}$ of the goals. What fraction of the goals did these two players score altogether?

4. Katrina likes to refinish furniture. This week she put two coats of varnish on a table. She used $\frac{1}{3}$ of a can of varnish for the first coat and $\frac{1}{4}$ of a can for the second coat. What fraction of a can did Katrina use altogether?

5. The gas tank in Mr. Yakimchuk's car was $\frac{7}{8}$ full when he left home. When he got to the cottage, the tank was $\frac{1}{8}$ full. How much gas did Mr. Yakimchuk use going to the cottage?

6. It took Lori $\frac{3}{4}$ h to walk from the library to the post office. It took her older sister $\frac{1}{4}$ h to walk the same route. How much longer did it take Lori to walk?

7. Jagdeep's younger brother, Harpreet, had a small garden. Harpreet planted $\frac{1}{4}$ of his garden with carrots. What fraction of his garden did he leave for other plants?

8. After the movie, Jana had $\frac{1}{2}$ of a bag of popcorn left. She ate popcorn during the bus ride home. If she arrived home with $\frac{1}{3}$ of a bag of popcorn, how much popcorn did she eat on the bus?

Check your answers by turning to the Appendix, page 155.

With practice, you will discover that you can mentally compute the sum of two fractions. This is especially true if the denominators are the same or if one denominator is a multiple of the other.

9. Mentally compute each of the following sums.

a. $\frac{1}{5}+\frac{2}{5}$ **b.** $\frac{1}{10}+\frac{6}{10}$ **c.** $\frac{1}{6}+\frac{4}{6}$

d. $\frac{1}{8}+\frac{1}{2}$ **e.** $\frac{2}{3}+\frac{1}{6}$ **f.** $\frac{2}{5}+\frac{3}{10}$

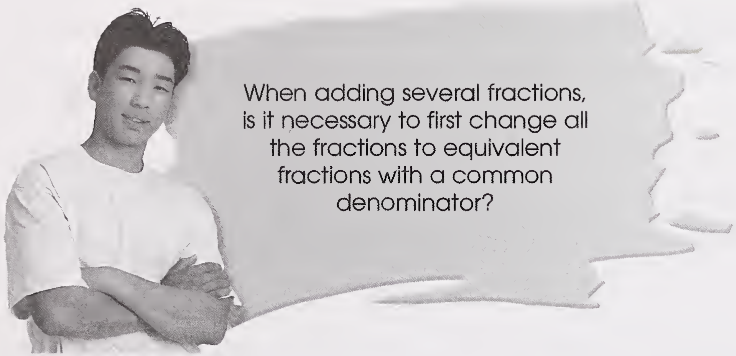
g. $\begin{array}{r} \frac{3}{4} \\ +\ \frac{1}{8} \\ \hline \end{array}$ **h.** $\begin{array}{r} \frac{1}{2} \\ +\ \frac{3}{8} \\ \hline \end{array}$ **i.** $\begin{array}{r} \frac{5}{6} \\ +\ \frac{1}{12} \\ \hline \end{array}$

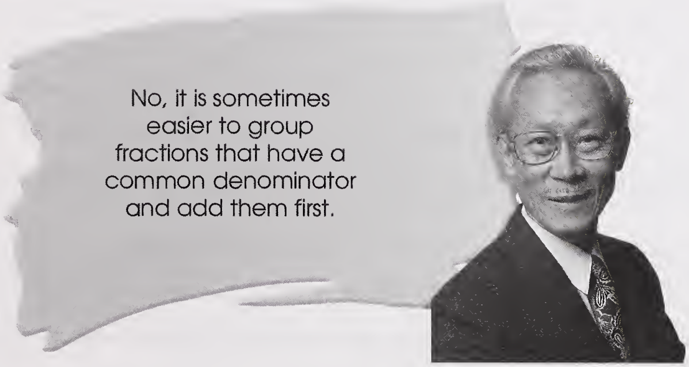
10. Mentally compute each of the following differences.

a. $\frac{7}{8}-\frac{1}{8}$ **b.** $\frac{3}{4}-\frac{1}{2}$ **c.** $\frac{1}{2}-\frac{1}{4}$

d. $\begin{array}{r} \frac{2}{5} \\ -\ \frac{1}{10} \\ \hline \end{array}$ **e.** $\begin{array}{r} \frac{7}{12} \\ -\ \frac{1}{6} \\ \hline \end{array}$ **f.** $\begin{array}{r} \frac{5}{6} \\ -\ \frac{2}{3} \\ \hline \end{array}$

Check your answers by turning to the Appendix, page 156.





Example

Evaluate the expression $\frac{2}{3}+\frac{1}{4}+\frac{1}{3}+\frac{1}{4}$.

Solution

$$\begin{aligned}\frac{2}{3}+\frac{1}{4}+\frac{1}{3}+\frac{1}{4} &= \left(\frac{2}{3}+\frac{1}{3}\right)+\left(\frac{1}{4}+\frac{1}{4}\right)\\ &= \frac{3}{3}+\frac{2}{4}\\ &= 1+\frac{1}{2}\\ &= 1\frac{1}{2}\end{aligned}$$

Grouping makes it much easier to complete the answer mentally.

11. Evaluate each of the following expressions. **Hint:** Group fractions that have a common denominator and add them first.

a. $\frac{1}{2}+\frac{2}{5}+\frac{3}{5}+\frac{3}{4}$

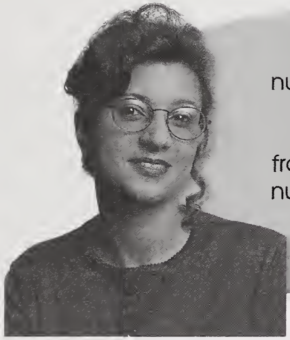
b. $\frac{1}{3}+\frac{1}{6}+\frac{1}{3}+\frac{1}{2}$

c. $\frac{1}{2}+\frac{3}{4}+\frac{1}{2}+\frac{7}{8}$

d. $\frac{2}{5}+\frac{1}{5}+\frac{3}{10}+\frac{2}{5}$

Check your answers by turning to the Appendix, page 157.

Adding Mixed Numbers



When you add mixed numbers vertically, line up the fractions and the whole numbers. First, add the fractions; then add the whole numbers. Regrouping may be required.

Example

The children at the summer camp spent $1\frac{1}{2}$ h playing games and $2\frac{1}{4}$ h making crafts. How much time did the children spend on these two activities?

Solution

Step 1: Line up the mixed numbers, and add the fraction parts. If necessary, change the fraction parts to equivalent fractions with a common denominator.

$$\begin{array}{r} 1\frac{1}{2} \\ +\ 2\frac{1}{4} \\ \hline \end{array} \quad \Rightarrow \quad \begin{array}{r} 1\ \frac{2}{4} \\ +\ 2\ \frac{1}{4} \\ \hline \frac{3}{4} \end{array}$$

Step 2: Add the whole number parts.

$$\begin{array}{r} 1\frac{1}{2} \\ +\ 2\frac{1}{4} \\ \hline \end{array} \quad \Rightarrow \quad \begin{array}{r} 1\ \frac{2}{4} \\ +\ 2\ \frac{1}{4} \\ \hline 3\ \frac{3}{4} \end{array}$$

The children spent $3\frac{3}{4}$ h on these two activities.

Example

Gina walked her dog for $2\frac{1}{2}$ h on Monday and for $1\frac{2}{3}$ h on Tuesday. How long did Gina walk her dog in total for these two days?

Solution

Step 1: Line up the mixed numbers, and add the fraction parts. If necessary, change the fraction parts to equivalent fractions with a common denominator.

$$\begin{array}{r} 2\frac{1}{2} \\ +\ 1\frac{2}{3} \\ \hline \end{array} \quad \Rightarrow \quad \begin{array}{rr} 2 & \frac{3}{6} \\ +\ 1 & \frac{4}{6} \\ \hline & \frac{7}{6} \end{array} \quad \Rightarrow \quad \begin{array}{rr} \overset{1}{2} & \frac{3}{6} \\ +\ 1 & \frac{4}{6} \\ \hline & \frac{1}{6} \end{array}$$

This step can be done mentally.

Regrouping is required, changing $\frac{7}{6}$ into $1\frac{1}{6}$.

Step 2: Add the whole number parts.

$$\begin{array}{r} 2\frac{1}{2} \\ +\ 1\frac{2}{3} \\ \hline \end{array} \quad \Rightarrow \quad \begin{array}{rr} \overset{1}{2} & \frac{3}{6} \\ +\ 1 & \frac{4}{6} \\ \hline 4 & \frac{1}{6} \end{array}$$

Gina walked her dog a total of $4\frac{1}{6}$ h.

12. Evaluate each of the following expressions.

a. $\begin{array}{r} 3\frac{1}{10} \\ +\ 1\frac{3}{5} \\ \hline \end{array}$ **b.** $\begin{array}{r} 4\frac{1}{6} \\ +\ 1\frac{2}{3} \\ \hline \end{array}$

c. $\begin{array}{r} 5\frac{5}{6} \\ +\ 1\frac{5}{12} \\ \hline \end{array}$ **d.** $\begin{array}{r} 1\frac{3}{4} \\ +\ 1\frac{1}{2} \\ \hline \end{array}$

13. In a single year, one charity raised $\$10\frac{1}{2}$ million and another raised $\$8\frac{1}{3}$ million. How much money was collected altogether by the two charities?

Check your answers by turning to the Appendix, page 158.

You can also add mixed numbers by first changing the mixed numbers to improper fractions, then adding the improper fractions as you would proper fractions.

Example

Marcus read $3\frac{3}{4}$ books last month. Sue read $1\frac{1}{2}$ books during the same period. How many books did the two students read last month?

Solution

$$3\frac{3}{4}+1\frac{1}{2}=\frac{15}{4}+\frac{3}{2}$$ ← Change the mixed numbers to improper fractions.

$$=\frac{15}{4}+\frac{6}{4}$$ ← Find equivalent fractions with a common denominator.

$$=\frac{15+6}{4}$$

$$=\frac{21}{4}$$

$$=5\frac{1}{4}$$

The two students read $5\frac{1}{4}$ books last month.

14. Add the following fractions by changing to improper fractions first.

a. $1\frac{7}{8}+2\frac{3}{4}$ **b.** $1\frac{1}{5}+2\frac{3}{10}$ **c.** $1\frac{2}{3}+3\frac{3}{4}$

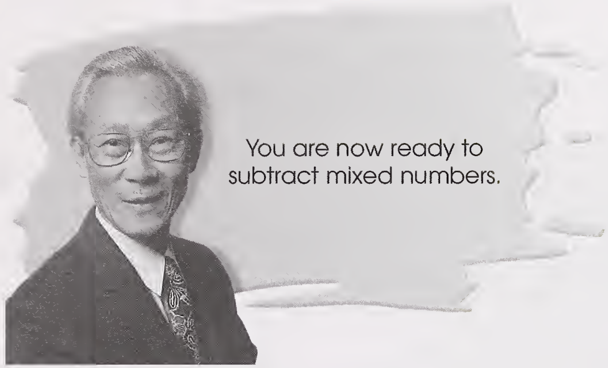
15. Mrs. Crowell enjoys gardening. One week, she transplanted $1\frac{3}{4}$ dozen tulips. The next week, she transplanted $1\frac{1}{2}$ dozen tulips. How many dozen tulips did Mrs. Crowell transplant altogether?

16. Franscesca enjoys her part-time job at the greenhouse. On Monday, she worked $1\frac{1}{2}$ h. On Tuesday, she worked another $2\frac{3}{4}$ h. Altogether, how much time did Franscesca work in the greenhouse on these two days?

Note: This is the problem you considered at the beginning of this activity.

Check your answers by turning to the Appendix, page 158.

Subtracting Mixed Numbers



When subtracting mixed numbers vertically, line up the fraction parts and the whole numbers. If necessary, change the fraction parts to equivalent fractions with a common denominator. Subtract the fraction parts and then subtract the whole number parts. Regrouping may be required.

Example

Maria practises the violin for $2\frac{1}{2}$ h each day. If she practised for $1\frac{1}{4}$ h in the morning, how much practice time is left that day?

Solution

Step 1: Line up the mixed numbers, and subtract the fraction parts. If necessary, change the fraction parts to equivalent fractions with a common denominator.

$$\begin{array}{r} 2\frac{1}{2} \\ -\ 1\frac{1}{4} \\ \hline \end{array} \quad \Rightarrow \quad \begin{array}{r} 2\ \frac{2}{4} \\ -\ 1\ \frac{1}{4} \\ \hline \frac{1}{4} \end{array}$$

$\frac{1}{2} = \frac{2}{4}$

No regrouping is required.

Step 2: Subtract the whole number parts.

$$\begin{array}{r} 2\frac{1}{2} \\ -\ 1\frac{1}{4} \\ \hline \end{array} \quad \Rightarrow \quad \begin{array}{r} 2\ \frac{2}{4} \\ -\ 1\ \frac{1}{4} \\ \hline 1\ \frac{1}{4} \end{array}$$

Maria has $1\frac{1}{4}$ h of practice time left.

Example

Robert has a summer job mowing lawns. He spent $4\frac{1}{4}$ h mowing lawns on Monday and $2\frac{1}{2}$ h mowing lawns on Tuesday. How much longer did he work on Monday than on Tuesday?

Solution

Step 1: Line up the mixed numbers, and subtract the fraction parts. If necessary, change the fraction parts to equivalent fractions with a common denominator.

$$\begin{array}{r} 4\frac{1}{4} \\ -\ 2\frac{1}{2} \\ \hline \end{array} \rightarrow \begin{array}{r} 4\frac{1}{4} \\ -\ 2\frac{2}{4} \\ \hline \end{array} \rightarrow \begin{array}{r} 3\ \frac{5}{4} \\ -\ 2\ \frac{2}{4} \\ \hline \frac{3}{4} \end{array}$$

Regrouping is required to change $4\frac{1}{4}$ into $3\frac{5}{4}$.

Step 2: Subtract the whole number parts.

$$\begin{array}{r} 4\frac{1}{4} \\ -\ 2\frac{1}{2} \\ \hline \end{array} \rightarrow \begin{array}{r} 4\frac{1}{4} \\ -\ 2\frac{2}{4} \\ \hline \end{array} \rightarrow \begin{array}{r} 3\ \frac{5}{4} \\ -\ 2\ \frac{2}{4} \\ \hline 1\ \frac{3}{4} \end{array}$$

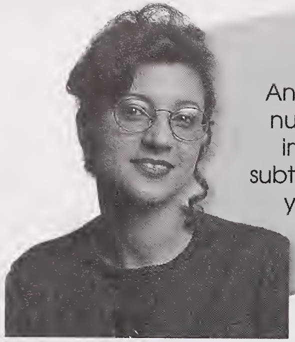
Robert worked $1\frac{3}{4}$ h longer on Monday than on Tuesday.

17. Use the vertical method to evaluate each of the following expressions.

a. $\begin{array}{r} 3\frac{1}{2} \\ -\ 1\frac{1}{4} \\ \hline \end{array}$ **b.** $\begin{array}{r} 4\frac{3}{4} \\ -\ 1\frac{1}{2} \\ \hline \end{array}$ **c.** $\begin{array}{r} 5\frac{1}{8} \\ -\ 2\frac{3}{4} \\ \hline \end{array}$ **d.** $\begin{array}{r} 6\frac{1}{4} \\ -\ 3\frac{5}{8} \\ \hline \end{array}$

18. Tiffany and Gerhardt have part-time jobs working at a skate rental booth. Last week, Tiffany worked for $9\frac{1}{2}$ h and Gerhardt worked for $8\frac{3}{4}$ h. How much longer did Tiffany work than Gerhardt?

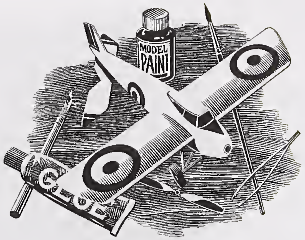
Check your answers by turning to the Appendix, page 159.



Another way to subtract mixed numbers is to change them to improper fractions first. Then subtract the improper fractions as you would proper fractions.

Example

Karen worked on a model airplane $3\frac{1}{2}$ h on Friday and $2\frac{3}{4}$ h on Saturday. How many more hours did she work on Friday than on Saturday?



Solution

$$3\frac{1}{2}-2\frac{3}{4}=\frac{7}{2}-\frac{11}{4}$$ ← Change the mixed numbers to improper fractions.

$$=\frac{14}{4}-\frac{11}{4}$$ ← Find equivalent fractions with a common denominator.

$$=\frac{3}{4}$$

Karen worked $\frac{3}{4}$ h more on Friday than on Saturday.

19. Subtract the following fractions by changing to improper fractions first.

a. $5\frac{7}{8}-2\frac{3}{4}$ **b.** $4\frac{2}{5}-2\frac{7}{10}$ **c.** $5\frac{1}{6}-3\frac{7}{12}$

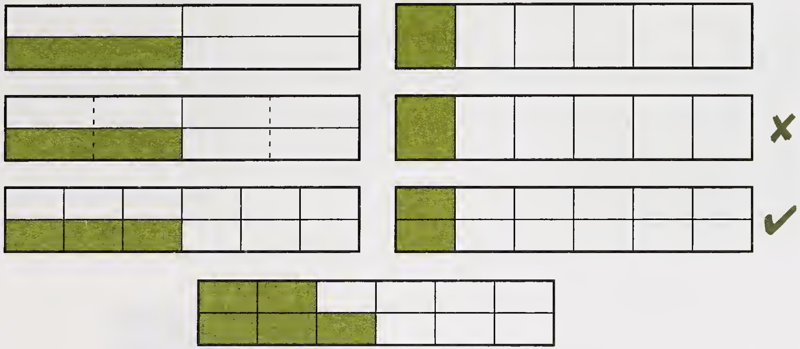
20. George had 4 chairs to paint. If he painted $1\frac{1}{2}$ chairs on Saturday, how many must he paint to complete the job?

Check your answers by turning to the Appendix, page 159.

Looking Back

In this activity, you added and subtracted fractions and mixed numbers. You continued to solve problems and developed your problem-solving skills. You saw that a model may be useful and aid in the understanding of fractions.

A student modelled an addition problem as follows.



21. In your journal, answer the following questions using the student's model.

  a. Which mathematical expression was the student trying to sum?

  b. Why did the student put an "X" by the second diagram?

  c. Explain whether or not it is always possible to find a common denominator between two fractions.

  d. One of your classmates tells you that $\frac{1}{2}+\frac{1}{2}=\frac{2}{4}$. Explain how that is an incorrect solution. Use two different methods to illustrate the correct answer.

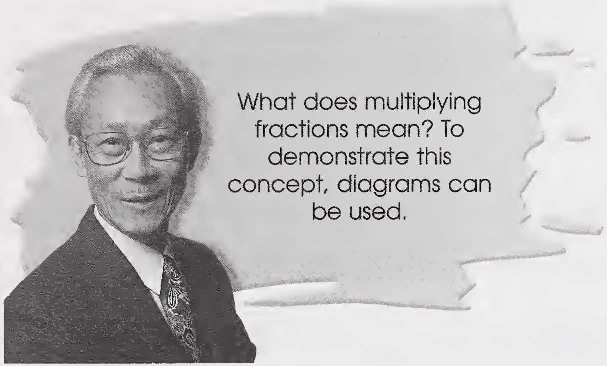
Check your answers by turning to the Appendix, page 160.

Activity 3: Multiplying Fractions and Mixed Numbers

A recipe calls for $4\frac{1}{2}$ cups of medium carrots. How many carrots will be required for twice the recipe? How many will be needed for half the recipe?

By the end of this activity, you will be able to solve problems like this.

Multiplying Fractions



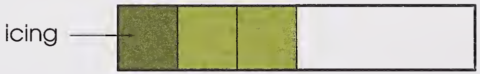
Example

A chocolate cake is partially iced. One-half of the chocolate cake is left over. Of the remaining cake, only $\frac{1}{3}$ has icing. The half of the cake eaten had no icing. How much of the cake had icing?

Solution

You can begin by modelling the $\frac{1}{2}$ of the cake that is left over.

Now, from this $\frac{1}{2}$, you know $\frac{1}{3}$ has icing. You need to divide the $\frac{1}{2}$ into thirds.



To see what this fraction is of the total amount, divide the other $\frac{1}{2}$ into thirds.

You can see that $\frac{1}{3}$ of $\frac{1}{2}$ is $\frac{1}{6}$.

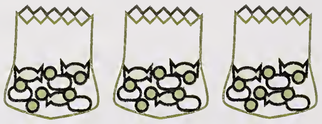
The mathematical sentence is $\frac{1}{2}\times\frac{1}{3}=\frac{1}{6}$.

Example

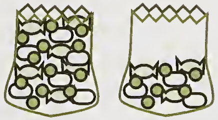
Three children each buy $\frac{1}{2}$ a bag of candy. How much did the 3 children buy altogether?

Solution

To model the question, you can draw 3 bags of candy, each $\frac{1}{2}$-filled.



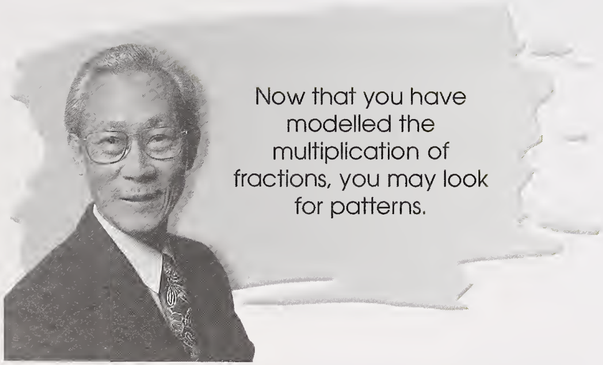
When you combine two bags, you will get one full bag.



The total amount is $\frac{3}{2}$ or $1\frac{1}{2}$ bags of candy.

The mathematical sentence is

$$3 \times \frac{1}{2} = \frac{3}{2}$$
$$= 1\frac{1}{2}$$



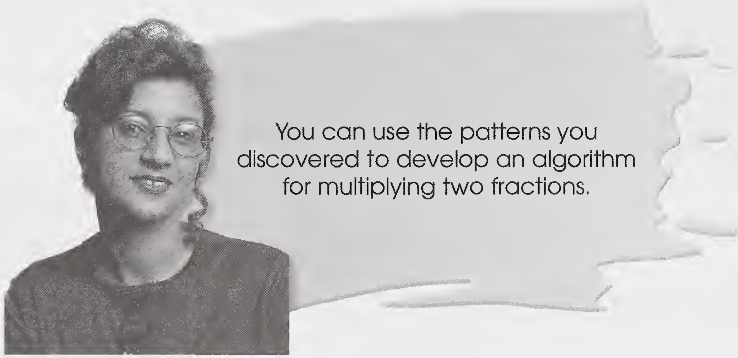
1. Use the given mathematical statements to answer the following questions.

$$\frac{1}{3} \times \frac{3}{4} = \frac{3}{12} = \frac{1}{4} \qquad \frac{2}{3} \times \frac{3}{4} = \frac{6}{12} = \frac{1}{2}$$

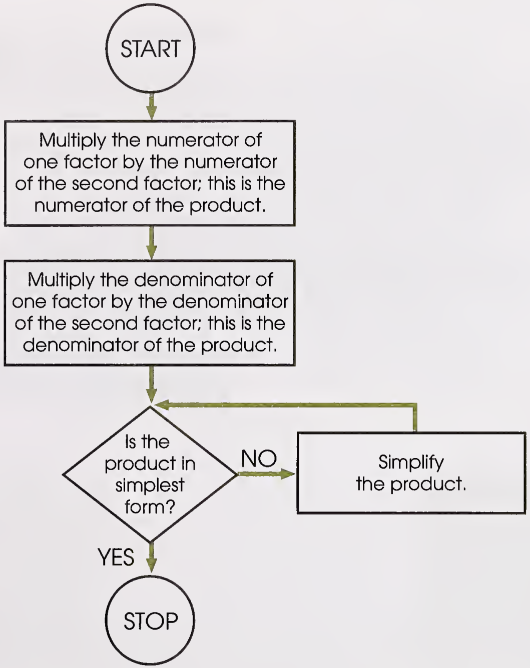
a. What relationship is there between the numerator of the answer (before it is simplified) and the numerators of the factors?

b. What relationship is there between the denominator of the answer (before it is simplified) and the denominators of the factors?

Check your answers by turning to the Appendix, page 160.



The algorithm for multiplying fractions is given in the following flow chart.



Notice how the algorithm is used in the following examples.

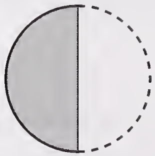
Example

There was $\frac{1}{2}$ of a cake in the refrigerator. Henri ate $\frac{2}{3}$ of this portion. What fraction of the whole cake did he eat?

Solution

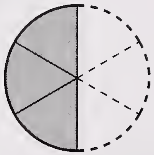
Pictures may help you visualize this situation.

This diagram shows 1 half of the cake.



The expression $\frac{2}{3} \times \frac{1}{2}$ means, “2 of 3 equal parts of 1 half is **how much** of the whole?”

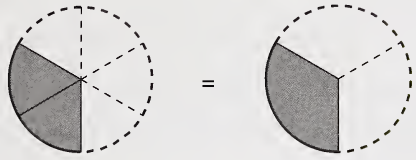
Because 1 half cannot be grouped into three equal parts, the fraction needs to be renamed as 3 sixths.



Ask yourself, “2 of 3 equal parts of 3 sixths is **how much** of the whole?”

The darker region in this diagram shows 2 of 3 equal parts of 3 sixths. This is the part Henri ate.

As the following diagram shows, the part Henri ate is **2 sixths**, or **1 third**, of the entire cake.



$\frac{2}{6} = \frac{1}{3}$

$$\frac{2}{3}\times\frac{1}{2}=\frac{2\times1}{3\times2}$$ ← Multiply the numerators. ← Multiply the denominators.

$$=\frac{2}{6}$$ ← Simplify.

$$=\frac{1}{3}$$

Henri ate $\frac{1}{3}$ of the whole cake.

Example

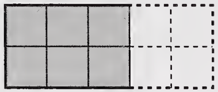
Sarah rototilled $\frac{3}{5}$ of her garden. Then she planted $\frac{1}{2}$ of the rototilled area. What fraction of the entire garden did Sarah plant?

Pictures may help you visualize the situation.

This diagram shows the part of the whole garden that was rototilled. It is 3 fifths of the whole garden.

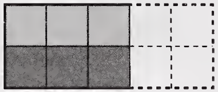
The expression $\frac{1}{2}\times\frac{3}{5}$ means, "1 of 2 equal parts of 3 fifths is **how much** of the whole?"

Because 3 fifths cannot be grouped into two equal parts, the fraction needs to be renamed 6 tenths.

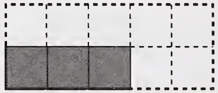


Ask yourself, "1 of 2 equal parts of 6 tenths is **how much** of the whole?"

The darker region in this diagram shows 1 of 2 equal parts of 6 tenths. This is the part that was planted.



As this diagram shows, Sarah planted **3 tenths** of the entire garden.



$$\frac{1}{2}\times\frac{3}{5}=\frac{1\times3}{2\times5}$$ ← Multiply the numerators. ← Multiply the denominators.

$$=\frac{3}{10}$$

Sarah planted $\frac{3}{10}$ of the garden.

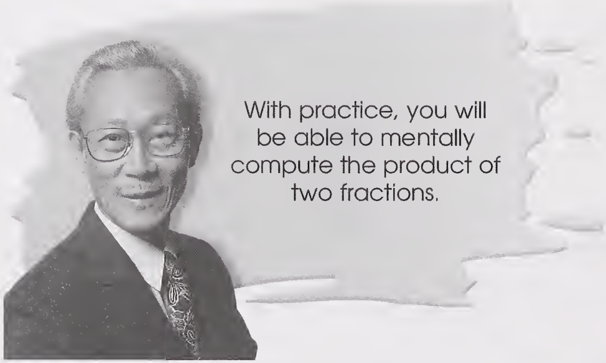
Use paper and pencil to answer questions 2 to 4.

2. Juanita mowed part of the lawn in the morning, leaving $\frac{3}{4}$ of the lawn unmowed. In the afternoon, she mowed $\frac{1}{2}$ of the unmowed area. What fraction of the entire lawn did Juanita mow in the afternoon?

3. Dale can walk from his home to the store in $\frac{3}{4}$ of an hour. He can ride his bike this distance in $\frac{1}{3}$ of that time. What fraction of an hour does it take Dale to ride his bike from his home to the store?

4. During lunch at a local restaurant, $\frac{2}{3}$ of the customers purchased salad. Of the people who bought salad, $\frac{3}{5}$ also ordered soup. What fraction of the customers had both soup and salad?

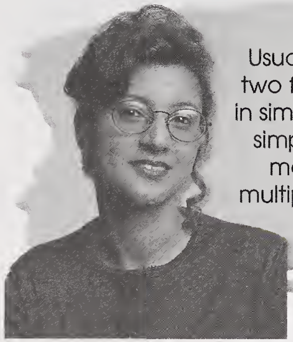
Check your answers by turning to the Appendix, page 160.



5. Mentally compute each of the following products.

a. $\frac{1}{3} \times \frac{1}{2}$ b. $\frac{3}{4} \times \frac{3}{5}$ c. $\frac{2}{3} \times \frac{4}{5}$

Check your answers by turning to the Appendix, page 161.



Usually, the product of two factors is expressed in simplest form. To make simplifying easier, you may divide before multiplying. This is called **cancelling**.

In multiplication, cancelling is the process of dividing a numerator and a denominator by a common factor.

Example

Phillipe gave Juan $\frac{1}{2}$ of his chocolate bar. If Juan ate $\frac{2}{3}$ of the piece Phillipe gave him, what fraction of the original chocolate bar did Juan eat?

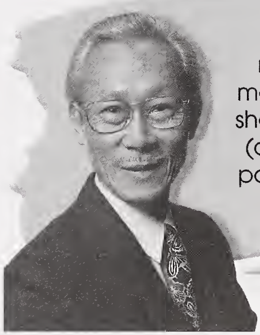
Solution

$$\begin{aligned}\frac{1}{2} \times \frac{2}{3} &= \frac{1 \times 2}{2 \times 3} \\ &= \frac{2 \times 1}{2 \times 3} \\ &= \frac{\overset{1}{\cancel{2}} \times 1}{\underset{1}{\cancel{2}} \times 3} \quad \longleftarrow \text{Simplify by dividing the numerator and the denominator by 2.} \\ &= \frac{1}{3}\end{aligned}$$

$2 \div 2 = 1$

$\frac{2}{2} = \frac{1}{1}$

Juan ate $\frac{1}{3}$ of the original chocolate bar.



When you are multiplying three or more fractions, look for shortcuts by cancelling (dividing) as much as possible before finding the product.

Example

Emily had $\frac{3}{4}$ of a container of candies. Emily gave $\frac{2}{3}$ of her candies to Matthew. Matthew, in turn, gave $\frac{1}{2}$ of his candies to Joshua. What fraction of the container of candies did Joshua receive?

Solution

$$\begin{aligned}\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4} &= \frac{1\times\cancel{2}^{1}\times\cancel{3}^{1}}{\cancel{2}_{1}\times\cancel{3}_{1}\times 4}\\ &= \frac{1\times1\times1}{1\times1\times4}\\ &= \frac{1}{4}\end{aligned}$$

← Simplify by dividing the numerator and the denominator by 2 and then by 3.

$\frac{2}{2}=\frac{1}{1}$; $\frac{3}{3}=\frac{1}{1}$

Joshua received $\frac{1}{4}$ of the container of candies.

Note: In the preceding example, different combinations of factors in the numerator and the denominator may be cancelled. Here is another way to cancel.

$$\begin{aligned}\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4} &= \frac{1\times\cancel{2}^{1}\times\cancel{3}^{1}}{2\times\cancel{3}_{1}\times\cancel{4}_{2}}\\ &= \frac{1\times1\times1}{2\times1\times2}\\ &= \frac{1}{4}\end{aligned}$$

← Simplify by dividing the numerator and the denominator by 2 and then by 3.

$\frac{2}{4}=\frac{1}{2}$; $\frac{3}{3}=\frac{1}{1}$

6. Compute each of the following products. Cancel wherever possible.

   a. $\frac{7}{8}\times\frac{4}{5}\times\frac{5}{8}$
   b. $\frac{1}{4}\times\frac{3}{5}\times\frac{2}{3}$
   c. $\frac{3}{5}\times\frac{4}{9}\times\frac{5}{6}$
   d. $\frac{2}{3}\times\frac{5}{6}\times\frac{9}{10}$

7. Using mental math, find the product of each of the following expressions.

   a. $\frac{1}{2}\times\frac{2}{3}$
   b. $\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}$
   c. $\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}\times\frac{4}{5}$

8. What pattern do you notice in question 7?

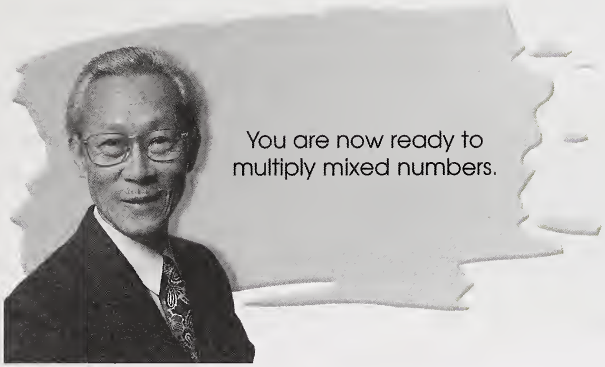
9. Use the pattern you discovered in question 8 to evaluate the following expression:

$$\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}\times\frac{4}{5}\times\ldots\times\frac{49}{50}$$

The three dots indicate that the factors between $\frac{4}{5}$ and $\frac{49}{50}$ are not shown, but the pattern of factors continues.

Check your answers by turning to the Appendix, page 161.

Multiplying Mixed Numbers



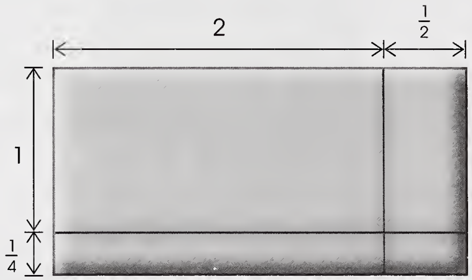
To multiply mixed numbers, change the mixed numbers to improper fractions and then multiply as you would proper fractions.

Example

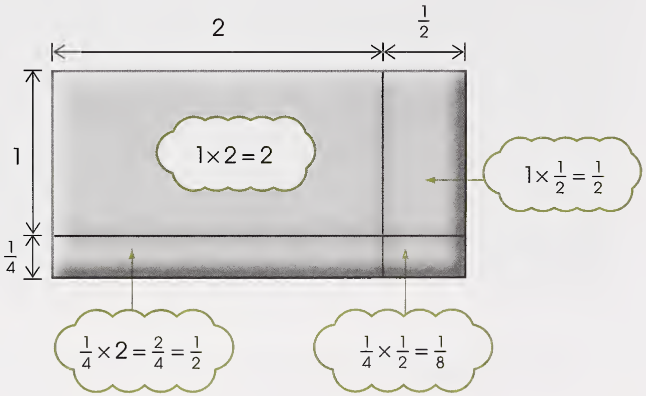
Kayla can skate $2\frac{1}{2}$ laps on the pond in 1 h. How many laps can she skate in $1\frac{1}{4}$ h?

Solution

Diagrams can help visualize the situation. The large rectangle represents $1\frac{1}{4} \times 2\frac{1}{2}$. Notice that there are four sections in the rectangle.



To find the product of $1\frac{1}{4}$ and $2\frac{1}{2}$, ask yourself, "What is the area of each section?" This can be calculated mentally. The following diagram shows the area of each section.



Now, ask yourself, "What is the total area of the four sections?"

$$2+\frac{1}{2}+\frac{1}{2}+\frac{1}{8}=2+\left(\frac{1}{2}+\frac{1}{2}\right)+\frac{1}{8}$$
$$=2+1+\frac{1}{8}$$
$$=3\frac{1}{8}$$

$$\therefore\ 1\frac{1}{4}\times 2\frac{1}{2}=3\frac{1}{8}$$

The problem in this example can also be solved by converting the mixed numbers to improper fractions and then multiplying.

$$1\frac{1}{4}\times 2\frac{1}{2}=\frac{5}{4}\times\frac{5}{2} \quad \longleftarrow \text{Change the mixed numbers to improper fractions.}$$
$$=\frac{5\times 5}{4\times 2}$$
$$=\frac{25}{8}$$
$$=3\frac{1}{8}$$

Kayla can skate $3\frac{1}{8}$ laps in $1\frac{1}{4}$ h.

10. Evaluate each of the following. Then use diagrams to illustrate each product.

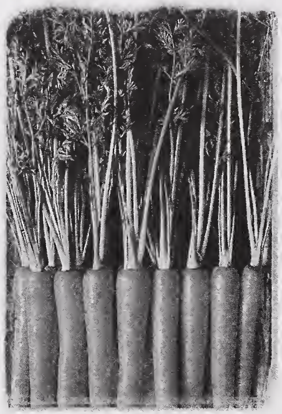
a. $1\frac{3}{5}\times 2\frac{1}{2}$ **b.** $2\frac{4}{5}\times 3\frac{1}{2}$ **c.** $1\frac{1}{6}\times 4\frac{2}{3}$

11. A recipe calls for $4\frac{1}{2}$ cups of medium carrots.

a. How many cups of carrots are needed for twice the recipe?

b. How many cups of carrots are needed for half the recipe?

Note: This is the problem you considered at the beginning of this activity.



Check your answers by turning to the Appendix, page 162.

Looking Back

In this activity, you multiplied fractions and mixed numbers. You continued to solve problems and develop problem-solving skills.

12. In your journal, solve the following problem:

A recipe for pancakes serves 4 people. It needs to be changed to serve 6 people.

You know that $\frac{6}{4}$ is equivalent to $\frac{3}{2}$ or $1\frac{1}{2}$. Therefore, you can assume that each ingredient has to be multiplied by $\frac{3}{2}$.

a. Determine the quantity required for each of the following ingredients. Use mixed numbers in your recipe.

- 1 egg
- $1\frac{1}{4}$ cups of milk
- $1\frac{1}{2}$ cups of flour
- 2 tbsp of sugar
- 3 tbsp of shortening
- $\frac{1}{2}$ tsp of salt
- $3\frac{1}{2}$ tsp of baking powder

b. If you needed to serve 15 people, what fraction would you multiply each ingredient by?

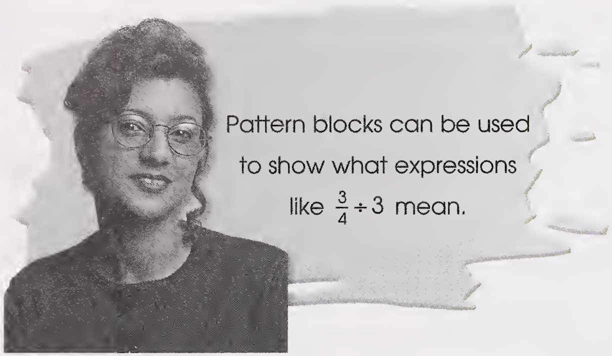
Check your answers by turning to the Appendix, page 164.

Activity 4: Dividing Fractions and Mixed Numbers

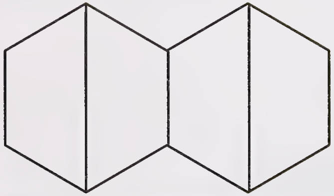
In the summer, Roger's father commutes to the office from his cottage. The family car uses $\frac{1}{4}$ of a tank of gas to travel from the cottage to the office. If the car has $\frac{3}{4}$ of a tank of gas, how many trips between his cottage and the office can Roger's father make?

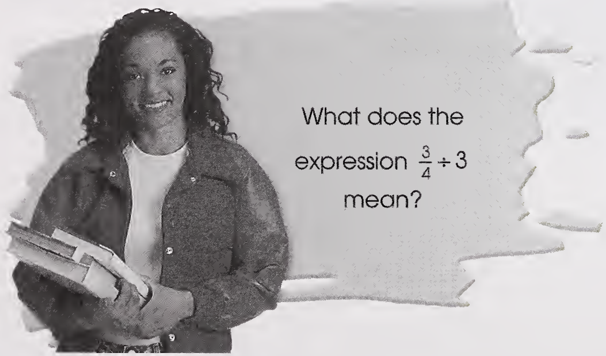
By the time you finish this activity, you will be able to solve problems like this.

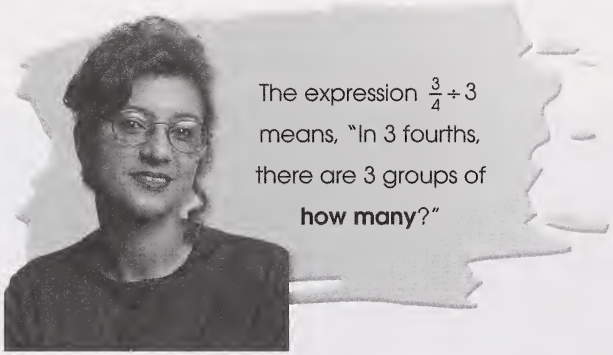
Suppose you need to divide a fraction by a whole number. Divide $\frac{3}{4}$ by 3.

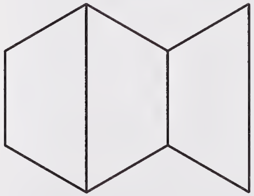


These blocks represent one whole.







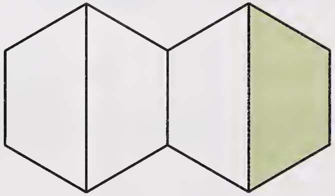


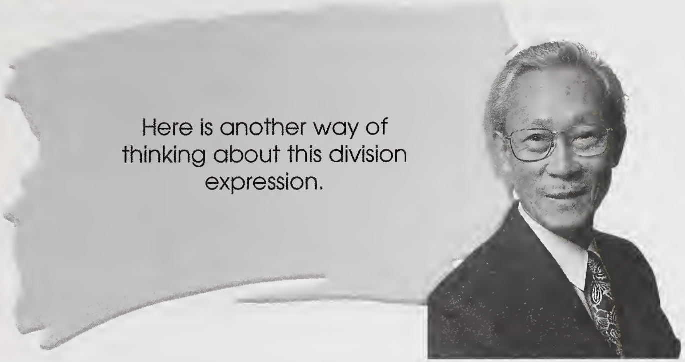
These blocks represent $\frac{3}{4}$ of a whole.

Dividing these into 3 smaller blocks give you this block.

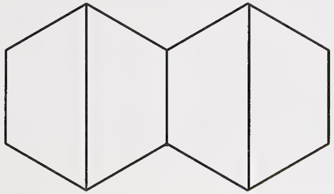
This is $\frac{1}{4}$ of a whole (as shown in the following diagram). Therefore,

$\frac{3}{4} \div 3 = \frac{1}{4}$.

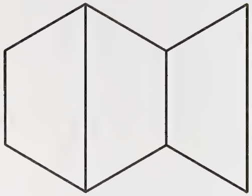




Illustrate by using pattern blocks. One whole is represented by these blocks.

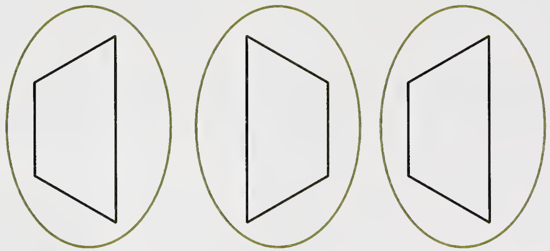


Step 1: Model $\frac{3}{4}$.



Remember: The expression $\frac{3}{4} \div 3$ means, "In 3 fourths, there are 3 groups of **how many**?"

Step 2: Divide the model into 3 equal parts.



In $\frac{3}{4}$, there are 3 groups of $\frac{1}{4}$. **Remember:** The block in each group represents $\frac{1}{4}$ of the whole.

$$\therefore \frac{3}{4} \div 3 = \frac{1}{4}$$

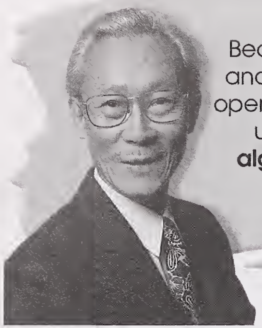
You can see in the diagram in Step 2 that 1 of the 3 equal parts of $\frac{3}{4}$ is $\frac{1}{4}$.

$$\therefore \frac{3}{4} \times \frac{1}{3} = \frac{1}{4}$$

You have just discovered that dividing $\frac{3}{4}$ by 3 produces the same result as multiplying $\frac{3}{4}$ by $\frac{1}{3}$.

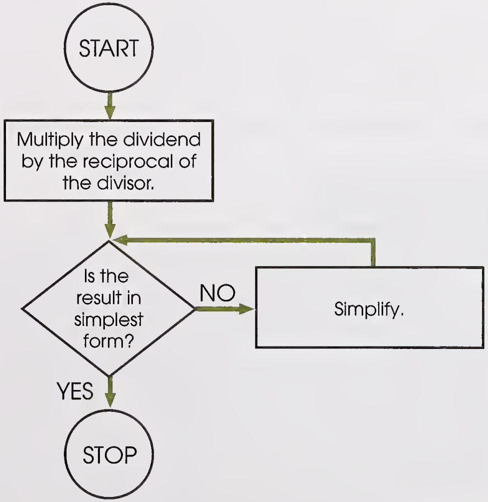
The operations of multiplication and division are inverse operations. The numbers 3 and $\frac{1}{3}$ are multiplicative inverses.

Inverse operations are operations that undo each other. Two numbers are multiplicative inverses if their product is 1. Multiplicative inverses are also called reciprocals.



Because multiplication and division are inverse operations, many people use the **reciprocal algorithm** for dividing fractions.

The reciprocal algorithm for dividing fractions is given in the following flow chart.



Work through the following examples, which use the reciprocal algorithm for dividing fractions.

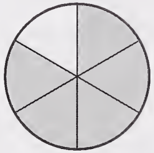
Example

Cordellia has $\frac{5}{6}$ of a pie. If 5 children share this remaining part of the pie, what fraction of the whole pie will each child receive?

Solution

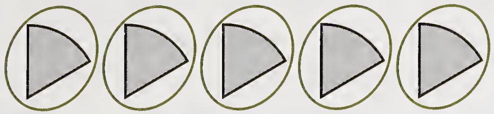
Method 1: Using Diagrams

The following diagram shows the fraction of the pie that will be shared by the 5 children. There is 5 sixths of a pie.



The expression $\frac{5}{6} \div 5$ can mean, "In 5 sixths, there are 5 groups of **how many**?"

As the following diagram shows, there are 5 groups of **1 sixth**.



Each child will receive $\frac{1}{6}$ of the whole pie.

Method 2: Paper and Pencil

$$\frac{5}{6} \div 5 = \frac{5}{6} \times \frac{1}{5} \quad \longleftarrow \text{Multiply the dividend by the reciprocal of the divisor.}$$
$$= \frac{\cancel{5} \times 1}{6 \times \cancel{5}}$$
$$= \frac{1}{6}$$

Each child will receive $\frac{1}{6}$ of the whole pie.

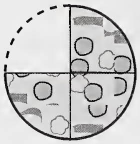
Example

Suki and Anneka share $\frac{3}{4}$ of a pizza. What fraction of the pizza will each girl receive?

Solution

Method 1: Using Diagrams

The following diagram shows the fraction of the pizza the 2 girls share (3 fourths of a pizza).



The expression $\frac{3}{4} \div 2$ can mean, "In 3 fourths, there are 2 groups of **how many**?"

In order to answer this question, you must rename 3 fourths as 6 eighths.

Ask yourself, "In 6 eighths, there are 2 groups of **how many**?"

As the following diagram shows, there are 2 groups of **3 eighths**.

Each girl will receive $\frac{3}{8}$ of the whole pizza.

Method 2: Using paper and Pencil

$$\frac{3}{4} \div 2 = \frac{3}{4} \times \frac{1}{2} \longleftarrow \text{Multiply the dividend by the reciprocal of the divisor.}$$
$$= \frac{3 \times 1}{4 \times 2}$$
$$= \frac{3}{8}$$

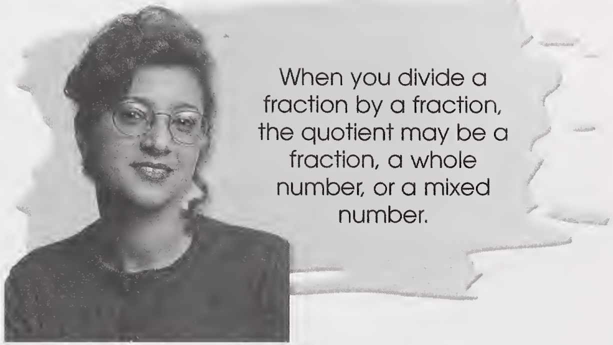
Each girl will receive $\frac{3}{8}$ of the whole pizza.

Use the reciprocal algorithm to answer questions 1 and 2.

1. Fred has $\frac{7}{12}$ of a carton of eggs. If he divides the eggs among 7 people, what part of the carton will each person receive?

2. Myrtle volunteers for $\frac{3}{4}$ h at a community hospital. If she divides her time equally doing 9 tasks, how much time will she spend on each task?

Check your answers by turning to the Appendix, page 165.



For example, $\frac{5}{12} \div \frac{7}{12} = \frac{5}{7}$; $\frac{5}{6} \div \frac{1}{12} = 10$; and $\frac{3}{4} \div \frac{1}{2} = 1\frac{1}{2}$.

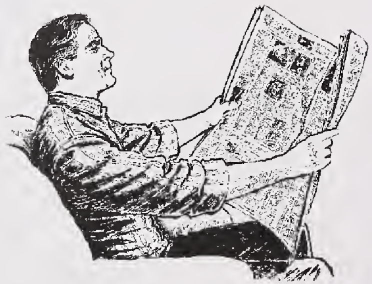
3. a. When is the quotient of two fractions less than 1?

   b. When is the quotient of two fractions greater than 1?

Check your answers by turning to the Appendix, page 165.

Example

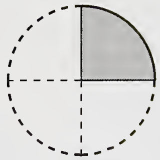
If Mr. Petrov can read one newspaper page in $\frac{1}{12}$ h, how many pages can he read in $\frac{1}{4}$ h?



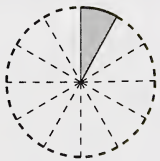
Solution

Method 1: Using Diagrams

The whole circle represents 1 hour. This diagram shows the time available to read. It is 1 fourth of an hour.

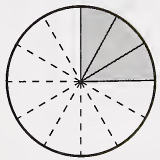


This diagram shows the time required by Mr. Petrov to read one page. It is 1 twelfth of an hour.



The expression $\frac{1}{4} \div \frac{1}{12}$ can mean, "**How many groups** of 1 twelfth are there in 1 fourth?"

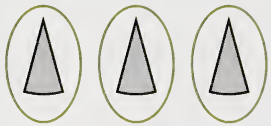
Because the fractions do not have a common denominator, 1 fourth must be renamed 3 twelfths.



Ask yourself, "**How many groups** of 1 twelfth are there in 3 twelfths?"

Since the divisor (1 twelfth) is less than the dividend (3 twelfths), there will be more than 1 group.

From the diagram, you can see that there are **3 groups** of 1 twelfth in 3 twelfths.



In $\frac{1}{4}$ h, Mr. Petrov can read 3 pages.

Method 2: Using Paper and Pencil

$$\begin{aligned}\frac{1}{4} \div \frac{1}{12} &= \frac{1}{4} \times \frac{12}{1}\\ &= \frac{1 \times \cancelto{3}{12}}{\cancelto{1}{4} \times 1}\\ &= \frac{3}{1}\\ &= 3\end{aligned}$$

In $\frac{1}{4}$ h, Mr. Petrov can read 3 pages.

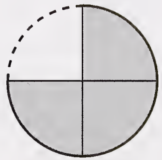
Example

Mr. Robertson gives music lessons. If each lesson takes $\frac{1}{2}$ h, how many lessons can he give in $\frac{3}{4}$ h?

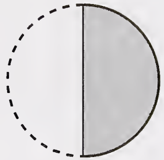
Solution

Method 1: Using Diagrams

This diagram shows the time available for lessons. It is 3 fourths of an hour.

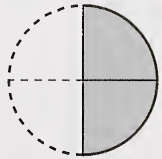


This diagram shows the time required for 1 lesson. It is 1 half of an hour.



The expression $\frac{3}{4} \div \frac{1}{2}$ can mean, "**How many groups** of 1 half are there in 3 fourths?"

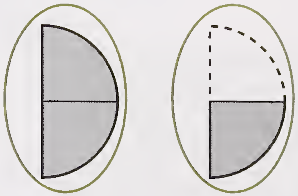
Because the fractions do not have a common denominator, the fraction 1 half must be renamed 2 fourths.



Ask yourself, "**How many groups** of 2 fourths are there in 3 fourths?"

Since the divisor, 2 fourths, is less than the dividend, 3 fourths, there will be more than 1 group.

From the diagram, you can see that there are $1\frac{1}{2}$ **groups** of 2 fourths in 3 fourths.



In $\frac{3}{4}$ h, Mr. Robertson can give $1\frac{1}{2}$ lessons.

Method 2: Using Paper and Pencil

$$\frac{3}{4} \div \frac{1}{2} = \frac{3}{4} \times \frac{2}{1}$$ ← Multiply the dividend by the reciprocal of the divisor.

$$= \frac{3 \times \cancel{2}^{1}}{\cancel{4}_{2} \times 1}$$

$$= \frac{3}{2}$$

$$= 1\frac{1}{2}$$

In $\frac{3}{4}$ h, Mr. Robertson can give $1\frac{1}{2}$ lessons.

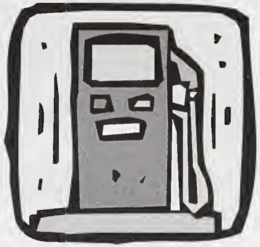
4. There is $\frac{5}{6}$ of a cake on a table. How many servings can be made from each of the following sizes?

a. $\frac{1}{6}$ of a cake

b. $\frac{1}{3}$ of a cake

c. $\frac{1}{4}$ of a cake

5. In the summer, Roger's father commutes to the office from his cottage. The family car uses $\frac{1}{4}$ of a tank of gas to travel from the cottage to the office. How many trips can Roger's father make between the cottage and the office with each of the following amounts of gas?



a. $\frac{3}{4}$ of a tank b. $\frac{1}{8}$ of a tank c. $\frac{1}{2}$ of a tank

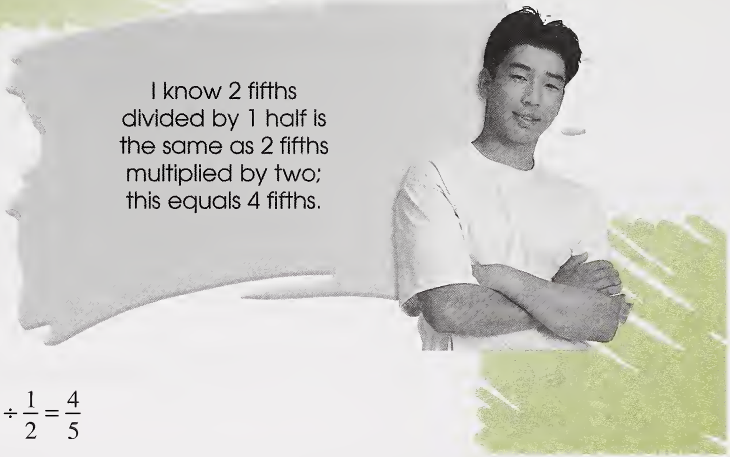
Note: You were introduced to question 5.a. at the beginning of this activity.

Check your answers by turning to the Appendix, page 165.

Example

Mentally divide $\frac{2}{5}$ by $\frac{1}{2}$.

Solution



$\therefore \frac{2}{5} \div \frac{1}{2} = \frac{4}{5}$

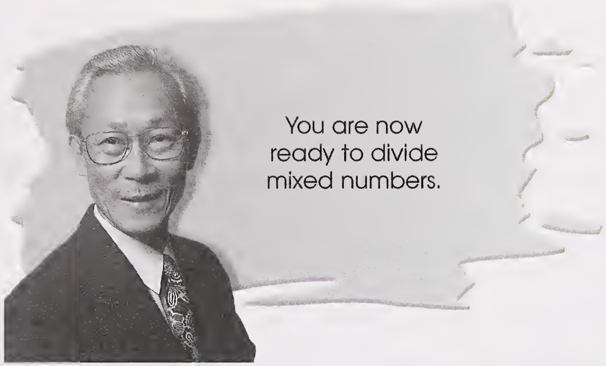
6. Mentally compute the quotient of each of the following expressions:

a. $\frac{5}{8} \div \frac{1}{8}$ b. $\frac{1}{3} \div \frac{2}{3}$ c. $\frac{2}{3} \div 2$

d. $\frac{4}{5} \div 2$ e. $\frac{3}{5} \div \frac{1}{2}$ f. $\frac{2}{3} \div \frac{1}{6}$

Check your answers by turning to the Appendix, page 166.

Dividing Mixed Numbers



In most cases, it is best to first change the mixed numbers to improper fractions. Then you can use the same process you use for dividing proper fractions.

Example

Haley and Cameron feed the chickens $3\frac{1}{3}$ pails of grain. If each feeder holds $1\frac{2}{3}$ pails of grain, how many feeders do they fill?

Solution

$$3\frac{1}{3} \div 1\frac{2}{3} = \frac{10}{3} \div \frac{5}{3} \quad \longleftarrow \text{Change mixed numbers to improper fractions.}$$

$$= \frac{10}{3} \times \frac{3}{5} \quad \longleftarrow \text{Multiply the dividend by the reciprocal of the divisor.}$$

$$= \frac{\overset{2}{\cancel{10}} \times \overset{1}{\cancel{3}}}{\underset{1}{\cancel{3}} \times \underset{1}{\cancel{5}}}$$

$$= \frac{2}{1}$$

$$= 2$$

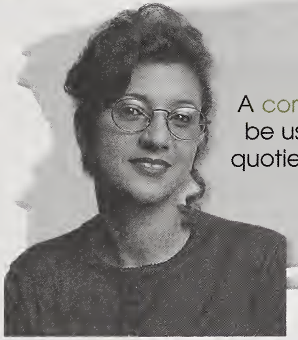
Haley and Cameron fill 2 feeders.

7. Evaluate each of the following expressions.

a. $2\frac{1}{2} \div \frac{5}{8}$ **b.** $6\frac{1}{2} \div 1\frac{3}{4}$ **c.** $8\frac{2}{3} \div 1\frac{1}{3}$

8. Orville is laying new tiles in his kitchen. If it takes him $1\frac{1}{4}$ minutes to glue down one tile, how many tiles can he glue down in $7\frac{1}{2}$ minutes?

Check your answers by turning to the Appendix, page 167.



A complex fraction can be used to express the quotient of two fractions.

A complex fraction has a fraction in the numerator and/or in the denominator.

The following are examples of complex fractions and their meanings:

- $\frac{\frac{1}{2}}{\frac{5}{6}}$ means $\frac{1}{2} \div \frac{5}{6}$
- $\frac{\frac{3}{4}}{\frac{2}{3}}$ means $\frac{3}{4} \div \frac{2}{3}$

Complex fractions can be simplified through division. Work through the following example.

Example

Simplify the complex fraction $\frac{\frac{1}{2}}{\frac{5}{6}}$.

Solution

$$\frac{\frac{1}{2}}{\frac{5}{6}} = \frac{1}{2} \div \frac{5}{6}$$ ← Rewrite the question using the + sign.

$$= \frac{1}{2} \times \frac{6}{5}$$ ← Multiply the dividend by the reciprocal of the divisor.

$$= \frac{1 \times \cancel{6}^{3}}{\cancel{2}_{1} \times 5}$$

$$= \frac{1 \times 3}{1 \times 5}$$

$$= \frac{3}{5}$$

9. Simplify each of the following complex fractions.

a. $\frac{\frac{2}{7}}{\frac{5}{7}}$ **b.** $\frac{\frac{1}{3}}{\frac{3}{4}}$ **c.** $\frac{\frac{3}{4}}{\frac{1}{2}}$

Check your answers by turning to the Appendix, page 167.

Now Try This

Use the following article to answer question 10.

QUADRUPEDIA GAZETTE

Section 3 **SPORTS**

Race for Four-Legged Championship

The finals of the 1-km race for the championship of the kingdom of Quadrupedia were held yesterday. Five animals entered the race: a bear, a giraffe, a rabbit, a dog, and a monkey. Dr. Do A. Little gave the signal to start the race.

The bear started right at the signal. It took him $\frac{1}{2}$ min longer to run the kilometre than the giraffe ran the kilometre.

The giraffe was daydreaming, so he started $\frac{5}{8}$ min after the bear. He ran the kilometre in $1\frac{3}{4}$ min.

The monkey ran twice as fast as the bear. He finished the race $\frac{1}{4}$ min before the giraffe.

The rabbit started $\frac{1}{3}$ min before the monkey and finished $\frac{1}{6}$ min before the giraffe.

The dog started $\frac{5}{12}$ min before the rabbit. He finished the race $1\frac{3}{4}$ min after the start of the race.

After the race the gold championship cup was awarded to the winner. [1]

[1] "Quadrupedia Gazette." Reprinted by permission from *SRA Mathematics Learning System* copyright 1974 by International Business Machines Corporation.

10. a. Which animal won the race? Which animal came in second? Which animal came in third? Which animal came in last?

b. Which animal ran the fastest? Which animal ran the second fastest? Which animal ran the third fastest? Which animal ran the slowest?

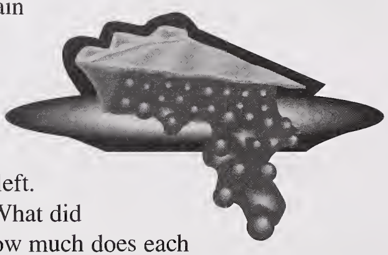
Check your answers by turning to the Appendix, page 168.

Looking Back

In this activity, you divided fractions and mixed numbers. You continued to solve problems and develop your problem-solving skills.

In your journal, do the following question.

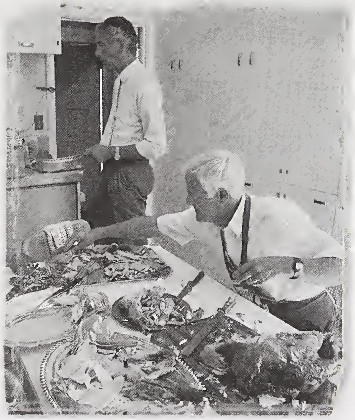
11. a. Use a model or diagram to explain why $\frac{1}{2} \div \frac{1}{4}$ is equal to 2.



b. A family of 4 was planning to equally share some leftover pie. There was 1 half of the pie left. The father said, "Let's half it." What did the father really want to say? How much does each person get?

Check your answers by turning to the Appendix, page 168.

Activity 5: Using a Calculator for Operations on Fractions and Mixed Numbers



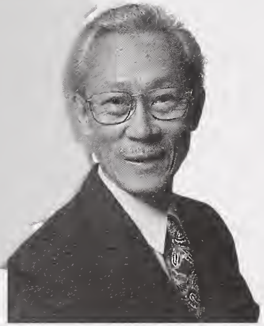
Andrew was cooking a special supper for five of his friends (and himself) and wanted to serve a special hamburger dish. The cookbook said he needed $\frac{1}{4}$ kg of hamburger for each guest. How much hamburger would he need for 6 people? He searched through the freezer and found two packages of hamburger: one weighed $\frac{3}{4}$ kg, and the other weighed $\frac{7}{8}$ kg. Does Andrew have enough hamburger for the recipe?

Andrew knew he could do the calculations on paper, but he wondered if using a calculator might be easier.

You have been calculating with fractions without using a calculator; and, often, that is the fastest and simplest way. Sometimes, though, you may wish to use a calculator.

Special care must be taken when working with fractions on your calculator. Some calculators have a fraction key, others have a pull down menu with an item called "▶ frac," which allows you to change a decimal to a fraction. You may need to consult your manual to learn how to work with fractions on your calculator.

Adding Fractions Using a Calculator



Fractions are usually not displayed in the standard manner on a calculator. The fraction $\frac{2}{3}$, for example, could be entered with these keystrokes.

$\boxed{2}\ \boxed{a\frac{b}{c}}\ \boxed{3}$

The display could show the following.

$2 \lrcorner 3.$

or

$2 \ulcorner 3.$

or

$2/3.$

The mixed number $1\frac{3}{4}$ could be entered as follows:

$\boxed{1}\ \boxed{a\frac{b}{c}}\ \boxed{3}\ \boxed{a\frac{b}{c}}\ \boxed{4}$

The display will show the following.

$1 \lrcorner 3 \lrcorner 4.$ **or** $1 \ulcorner 3 \ulcorner 4.$

Note: The mixed number $1\frac{3}{4}$ may be entered as an improper fraction, $\frac{5}{4}$.

$5 \lrcorner 4.$ **or** $5 \ulcorner 4.$

The following example shows you how to use the fraction key. You should first estimate the answer, since it is very easy to press a wrong key.

Example

Jordan is taking inventory in a stationery store. He finds three partial boxes of red pens. The first box is $\frac{1}{3}$ full; the second box is $\frac{2}{5}$ full; and the third box is $\frac{3}{4}$ full. How many full boxes of red pens can Jordan make?

Solution

Use a calculator to evaluate the expression $\frac{1}{3}+\frac{2}{5}+\frac{3}{4}$.

Step 1: Estimate the answer.

The sum of $\frac{1}{3}+\frac{2}{5}+\frac{3}{4}$ is between 1 and 3.

Step 2: Press the following key sequence.

$\boxed{1}\ \boxed{a\frac{b}{c}}\ \boxed{3}\ \boxed{+}\ \boxed{2}\ \boxed{a\frac{b}{c}}\ \boxed{5}\ \boxed{+}\ \boxed{3}\ \boxed{a\frac{b}{c}}\ \boxed{4}\ \boxed{=}$

$1 \lrcorner 29 \lrcorner 60.$

The calculator has calculated the answer as $1\frac{29}{60}$.

Step 3: Compare the calculated answer to the estimate.

Because $1\frac{29}{60}$ is between 1 and 3, the answer is reasonable.

Jordan can make $1\frac{29}{60}$ boxes of red pens.

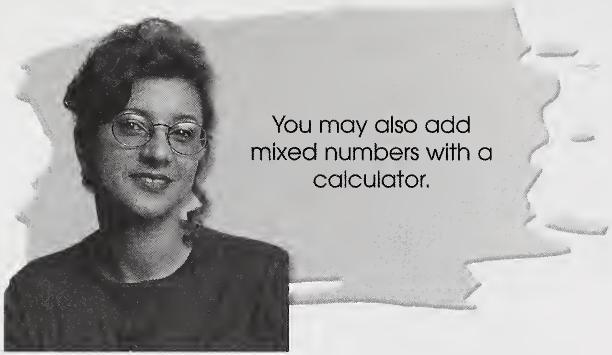
Use a scientific calculator with a fraction key to answer question 1.

1. Calculate the following sums.

   a. $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}$

   b. $\frac{2}{3}+\frac{3}{5}+\frac{5}{6}$

Check your answers by turning to the Appendix, page 169.



It is recommended that you always estimate the answer first, since it is easy to press a wrong key.

Example

Use a calculator to evaluate the expression $1\frac{1}{3}+2\frac{3}{4}$.

Solution

Step 1: Estimate the answer.

Rounding	Front-end Digits
$1\frac{1}{3}+2\frac{3}{4} \doteq 1+3$	$1\frac{1}{3}+2\frac{3}{4} \doteq 1+2$
$\doteq 4$	$\doteq 3$

Step 2: Press the following key sequence.

$\boxed{1}\ \boxed{a\frac{b}{c}}\ \boxed{1}\ \boxed{a\frac{b}{c}}\ \boxed{3}\ \boxed{+}$

$\boxed{2}\ \boxed{a\frac{b}{c}}\ \boxed{3}\ \boxed{a\frac{b}{c}}\ \boxed{4}\ \boxed{=}$

$\boxed{4 \lrcorner 1 \lrcorner 12.}$

Step 3: Compare the calculated answer with the estimate.

Rounding	Front-end Digits
$4\frac{1}{12} \doteq 4$	$4\frac{1}{12} \doteq 3$

The answer is reasonable.

$\therefore\ 1\frac{1}{3} + 2\frac{3}{4} = 4\frac{1}{12}$

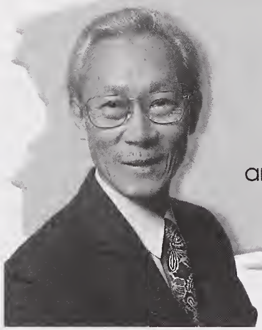
2. Complete the following using your calculator.

a. $6\frac{9}{13} + 2\frac{1}{5}$

b. $148\frac{1}{9} + 8\frac{5}{6}$

Check your answers by turning to the Appendix, page 170.

Subtracting Fractions Using a Calculator



You may use a calculator with a fraction key to subtract fractions and mixed numbers.

Example

Evaluate the expression $3\frac{1}{2} - 1\frac{3}{4}$.

Solution

Step 1: Estimate the answer.

Rounding	Front-end Digits
$3\frac{1}{2} - 1\frac{3}{4} \doteq 4 - 2$	$3\frac{1}{2} - 1\frac{3}{4} \doteq 3 - 1$
$\doteq 2$	$\doteq 2$

Step 2: Press the following key sequence.

3 $a\frac{b}{c}$ 1 $a\frac{b}{c}$ 2 − 1 $a\frac{b}{c}$ 3 $a\frac{b}{c}$ 4 =

1⌟3⌟4.

Step 3: Compare the calculated answer to the estimate.

Rounding	Front-end Digits
$1\frac{3}{4} \doteq 2$	$1\frac{3}{4} \doteq 2$

The answer is reasonable.

$\therefore\ 3\frac{1}{2} - 1\frac{3}{4} = 1\frac{3}{4}$

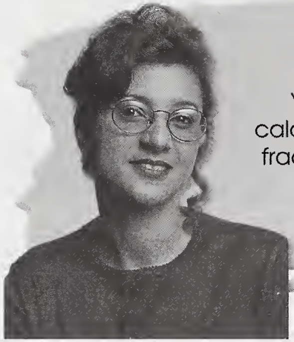
3. Complete the following problems using your calculator.

   a. Dave ran $2\frac{1}{2}$ km on Tuesday. On Wednesday, he ran $3\frac{1}{3}$ km. How much further did he run on Wednesday?

   b. Azra has $7\frac{1}{2}$ cups of flour. She will use $4\frac{3}{4}$ cups for one batch of bread. She also wants to make a cake that calls for $2\frac{1}{2}$ cups of flour. Will she have enough flour to make the cake?

Check your answers by turning to the Appendix, page 170.

Multiplying and Dividing Fractions and Mixed Numbers Using a Calculator



Example

Evaluate the expression $3\frac{7}{8} \times 4\frac{1}{3}$.

Solution

Step 1: Estimate the answer.

Rounding	Front-end Digits
$3\frac{7}{8} \times 4\frac{1}{3} \doteq 4 \times 4$	$3\frac{7}{8} \times 4\frac{1}{3} \doteq 3 \times 4$
$\doteq 16$	$\doteq 12$

Step 2: Press the following key sequence.

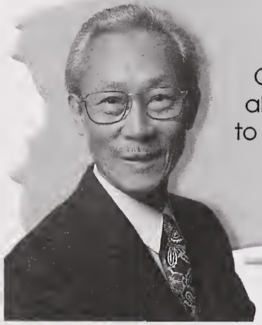
3 $a\frac{b}{c}$ 7 $a\frac{b}{c}$ 8 × 4 $a\frac{b}{c}$ 1 $a\frac{b}{c}$ 3 =

16 ⌟ 19 ⌟ 24.

Step 3: Compare the calculated answer to the estimate.

Because $16\frac{19}{24} \doteq 16$, the answer is reasonable.

$$\therefore\ 3\frac{7}{8} \times 4\frac{1}{3} = 16\frac{19}{24}$$



Of course, you can also use a calculator to divide fractions and mixed numbers.

Remember that in the paper-and-pencil method, dividing by a fraction is the same as multiplying by its reciprocal. You don't need to do this step when you use a calculator.

Example

Ms. Reichle is buying large tins of soup for the school cafeteria. The cafeteria sells soup in $1\frac{1}{2}$-cup servings. Each can makes 48 cups. How many servings are there from one can?

Solution

First, estimate the answer.

$$48 \div 1 = 48$$
$$48 \div 2 = 24$$

The answer will be between 24 and 48.

Now, divide 48 by $1\frac{1}{2}$.

4 8 ÷ 1 $a\frac{b}{c}$ 1 $a\frac{b}{c}$ 2 =

32

This result fits within the estimated values. Therefore, one can will provide 32 servings.

Example

Use a calculator to calculate $1\frac{5}{8} \div 3\frac{1}{2}$.

Solution

Estimate the answer.

$$2 \div 4 \doteq \frac{2}{4}$$
$$\doteq \frac{1}{2}$$

Enter the keystrokes.

1 $a\frac{b}{c}$ 5 $a\frac{b}{c}$ 8 ÷ 3 $a\frac{b}{c}$ 1 $a\frac{b}{c}$ 2 =

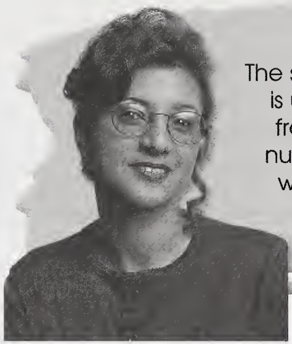
13 ⌟ 28

Compare the calculated answer to the estimate.

$$\frac{13}{28} \doteq \frac{1}{2}$$

The answer is reasonable.

$$\therefore\ 1\frac{5}{8} \div 3\frac{1}{2} = \frac{13}{28}$$



The same entry pattern is used for entering fractions or mixed numbers, no matter what operation is involved.

Use a scientific calculator to answer questions 4 and 5.

To practise using your calculator, do the following questions. First, estimate the answer; then do the calculation. Leave your answer either as a fraction in simplest form or as a mixed number.

4. a. $\begin{array}{r} 7\frac{7}{9} \\ +\ 1\frac{1}{3} \\ \hline \end{array}$

   b. $2\frac{7}{8} + 5\frac{1}{2}$

   c. $\begin{array}{r} 27\frac{2}{3} \\ -\ 6\frac{7}{9} \\ \hline \end{array}$

   d. $\frac{39}{85} - \frac{2}{15}$

   e. $6\frac{3}{5} \times 2\frac{7}{9}$

   f. $\frac{27}{10} \times 5\frac{1}{2}$

   g. $\frac{12}{25} \div \frac{4}{15}$

   h. $8\frac{2}{3} \div 2\frac{3}{5}$

5. When knitting, for example, the size of the knitting needles and the kind of yarn will affect the tension. Tension is measured by the number of stitches in a square that is $2\frac{1}{2}$ cm by $2\frac{1}{2}$ cm.

Frieda used two different brands of yarn but the same needles to knit two baby blankets. She then measured a $2\frac{1}{2}$-cm square from each afghan to compare the tension. She discovered the first square had $5\frac{1}{4}$ rows with $7\frac{1}{4}$ stitches per row. The second square had $5\frac{1}{2}$ rows with $4\frac{1}{2}$ stitches per row.

a. How many stitches were in the first square?

b. How many stitches were in the second square?

Check your answers by turning to the Appendix, page 171.

Looking Back

In this activity, you used your calculator to practise adding, subtracting, multiplying, and dividing fractions.

You know now that Andrew, the person you read about at the beginning of this activity, could use a calculator to see if he had enough hamburger for his recipe.

6. Using your calculator, solve the problem involving Andrew that was introduced at the beginning of this activity.

   a. How much hamburger does Andrew need if he is feeding 6 people and each serving of the recipe requires $\frac{1}{4}$-kg of hamburger?

   b. Will Andrew have enough hamburger to make the recipe if one package is $\frac{3}{4}$ kg and the second package is $\frac{7}{8}$ kg?

7. **a.** Describe a computation with fractions that might be done more efficiently using paper and pencil or mental math.

   b. Describe a computation with fractions that might be done more efficiently using a calculator.

Check your answers by turning to the Appendix, page 174.

Conclusion

This section helped to develop your sense for fractions, the different ways fractions can be expressed, and the sizes of various fractions and decimal numbers.

First, you found equivalent forms for fractions and decimal numbers, expressed numbers less than 1 as proper fractions and decimal numbers, expressed numbers greater than 1 as improper fractions and mixed numbers, and ordered fractions. You then practised adding, subtracting, multiplying, and dividing fractions using mental math, diagrams, and paper and pencil. Finally, you used a scientific calculator to carry out operations on fractions.

You were shown many situations, such as dividing up a pizza among friends or the total time in a week you walked your dog, where fractions and decimal numbers are used. Were you surprised to discover that fractions are used in so many places?

Remember, practising operations on fractions can make you skilled in using them to solve day-to-day problems relating to fractions.

Assignment

Turn to Assignment Booklet 2A and complete the assignment for Section 1.

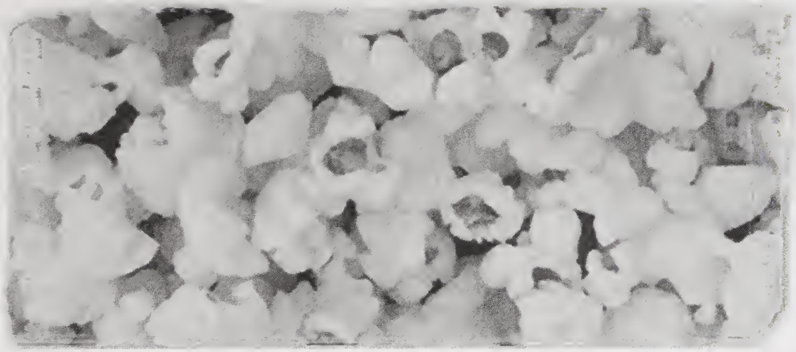
SECTION 2

Using Fractions: Rates, Ratios, and Proportions

The railroads played a vital role in the development of Western Canada. Historians remind us that this country was linked together with ribbons of steel. In the late nineteenth century and early twentieth century, trains brought settlers to the prairies, hauled grain to eastern ports for domestic consumption and export, and brought consumer goods back west. Throughout this period, freight rates—the charges levied on the goods hauled—provided revenue to the railroads. These rates varied depending on the commodity, points of origin and destination, size, and weight. Did you know that transportation and other related rates are often expressed as percentages or proportions of the value of the item shipped?

In this section, you will extend your knowledge of fractions to rates, ratios, proportions, and percents. Throughout this section, you will apply these skills to solve a variety of real-world problems.

Activity 1: Ratios



Did you know that the ratio of dried corn kernels to popped corn is 1 to 24? That is, if you pop 125 mL of dried corn kernels you will get 3000 mL of popped corn.

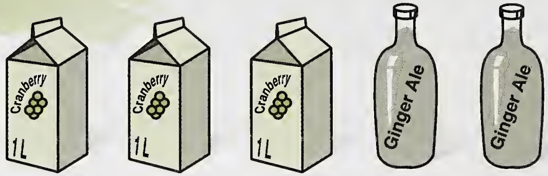
A ratio is a comparison that describes a number relationship between quantities expressed in the same unit or in units that can be converted to the same unit.

When you write a ratio, make sure you indicate what is being compared. Also, be careful of the order in which the numbers are given.

The unit of measurement may be a standard unit, such as metres, grams, litres, or hours. It may also be a non-standard unit, such as scoops or paces. The unit is unimportant as long as the units of the quantities being compared are the same.

Example

Maria made 5 L of punch by combining 3 L of cranberry juice and 2 L of ginger ale. What are the different ratios that can be used to describe this situation?



Solution

The different ratios are as follows:

- The ratio of the amount of cranberry juice to the amount of ginger ale is 3 to 2.
- The ratio of the amount of cranberry juice to the total amount of punch is 3 to 5.
- The ratio of the amount of ginger ale to the amount of cranberry juice is 2 to 3.
- The ratio of the amount of ginger ale to the total amount of punch is 2 to 5.

Work through the following problems involving ratios.

1. Franz is the tallest person in his family. His height is 199 cm. His father's height is 175 cm, and his mother's height is 167 cm.

   a. What is the ratio of Franz's height to his father's height?

   b. What is the ratio of Franz's height to his mother's height?

2. Mavis is 15 years old, and Emma is 14 years old.

   a. What is the ratio of Emma's age to Mavis's age?

   b. What is the ratio of Mavis's age to Emma's age?

Check your answers by turning to the Appendix, page 174.

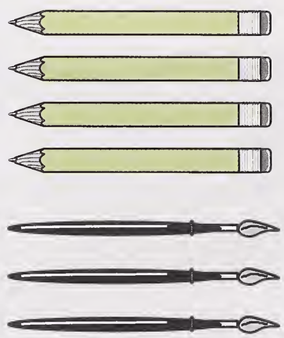
Different Forms

You discovered that fractions can be written in different forms: proper fractions can be written as decimal numbers, and improper fractions can be written as mixed numbers or decimal numbers.

Ratios can also be written in different forms. Three forms of ratios are shown in the next example.

Example

There are 4 pencils and 3 paintbrushes sitting on a desk. Write the ratio of the number of pencils to the number of brushes.



Solution

Method 1: Using the Word *to*

The ratio of the number of pencils to the number of brushes is 4 to 3.

Method 2: Using a Colon

The ratio of the number of pencils to the number of brushes is $4:3$.

This is read as "4 to 3."

Method 3: Using Fraction Form

The ratio of the number of pencils to the number of brushes is $\frac{4}{3}$.

This is read as "4 to 3."

The first number in a ratio is called the first term. The second number in a ratio is called the second term.

For question 3, express each ratio as a statement. Write each ratio three ways: using the word *to*, using a colon, and using fraction form.

3. The Hawks hockey team won 41 games, lost 10 games, and tied 8 games.

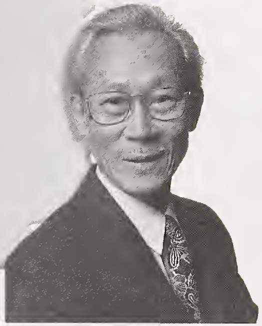
   a. Write the win-loss ratio.

   b. Write the ratio of the number of wins to the total number of games played.

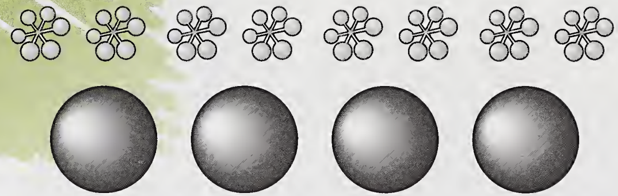
Check your answers by turning to the Appendix, page 174.

Lowest Terms

Sometimes it is desirable to write a ratio in lowest terms or simplest form. A ratio in simplest form has the lowest whole-number terms possible.



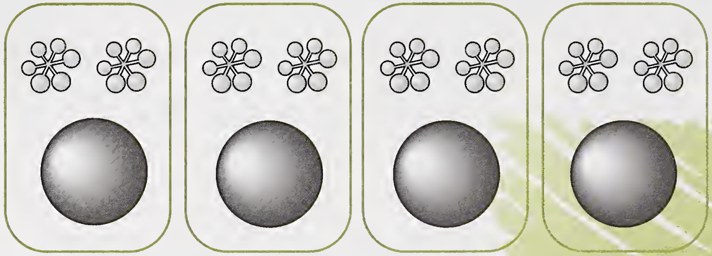
Example



What is the ratio of the number of jacks to the number of balls? Write the ratio in lowest terms.

Solution

The overall ratio is 8 to 4, but this is not in lowest terms (simplest form). Regroup the jacks and balls so that the groups are equal and that there are the fewest possible number of jacks and balls in each group.



As the diagram shows, there are 2 jacks for every 1 ball. So, the ratio of the number of jacks to the number of balls is 2 to 1.

You can think of a ratio as a describing rule or relationship. The ratio 2:1 reminds you there are 2 jacks for every ball. When you write a ratio in lowest terms, you are simplifying the rule. However, you are not changing the relationship of the numbers being compared.

Example

Maria has a mass of 45 kg, and Jean has a mass of 54 kg. What is the ratio of Maria's mass to Jean's mass? Express the ratio in lowest terms.

Solution

$$\frac{45}{54} = \frac{5}{6} \quad (\div 9)$$

$$\frac{\text{Maria s mass (kg)}}{\text{Jean s mass (kg)}}$$

The ratio of Maria's mass to Jean's mass is 5 to 6.

Express each of the ratios in questions 4 to 6 in lowest terms.

4. One tube of toothpaste contains 200 mL. A smaller tube of toothpaste contains 100 mL. What is the ratio of the amount of toothpaste in the larger tube to the amount of toothpaste in the smaller tube?

5. To make a concrete foundation, 300 kg of gravel is mixed with 450 kg of sand and 100 kg of cement.

   a. What is the ratio of the mass of gravel to the mass of sand?

   b. What is the ratio of the mass of gravel to the mass of cement?

6. At the 1988 Winter Olympics in Calgary, West Germany won 2 gold medals, 4 silver medals, and 2 bronze medals. Switzerland won 5 gold medals, 5 silver medals, and 5 bronze medals.

   a. What is the ratio of the number of gold medals won by West Germany to the total number of medals won by West Germany?

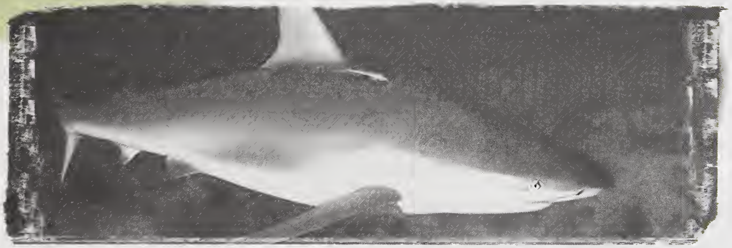
   b. What is the ratio of the number of gold medals won by Switzerland to the total number of medals won by Switzerland?

Check your answers by turning to the Appendix, page 174.

Remember, a ratio compares quantities measured in the **same** units. In some problems, you may have to convert quantities to the same unit before you can write the ratio.

Example

The length of a shark's longest tooth is 3 cm. The length of the shark's body is 3.3 m. What is the ratio of the length of the shark's longest tooth to the length of the shark's body?



ZEDCOR, INC.

Solution

Step 1: Write the measurements in the same unit. Express both in centimetres or both in metres. Converting both to centimetres will eliminate decimals in this case.

$$3.3 \text{ m} = 330 \text{ cm}$$

Step 2: Write the ratio in lowest terms.

$$\frac{3}{330} \overset{\div 3}{\underset{\div 3}{=}} \frac{1}{110}$$

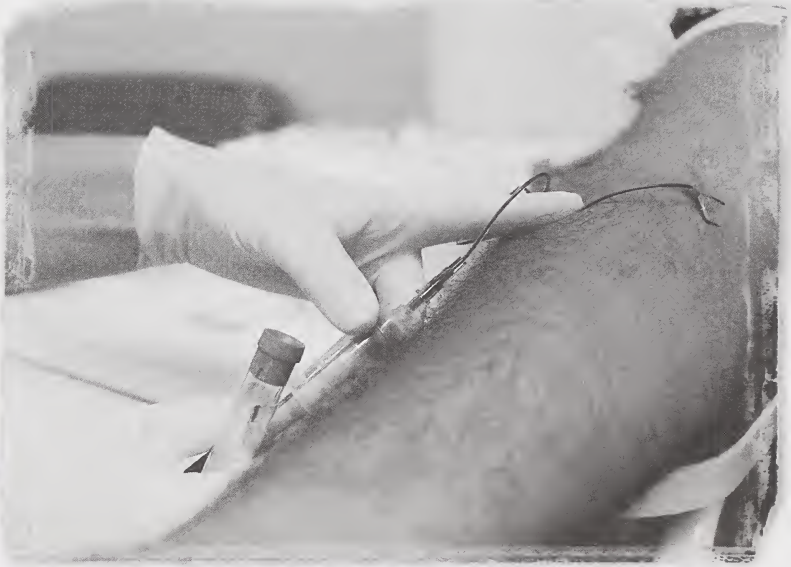
$$\frac{\text{length of longest tooth (cm)}}{\text{length of body (cm)}}$$

The ratio of the length of the shark's longest tooth to the length of the shark's body is 1 to 110.

Express each of the ratios in questions 7 and 8 in lowest terms.

7. Harvey has a pumpkin and a zucchini. The pumpkin has a mass of 2 kg and the zucchini has a mass of 250 g. What is the ratio of the mass of the pumpkin to the mass of the zucchini?

8. The body of an adult contains about 5 L of blood. A blood donor gives 450 mL of blood. What is the ratio of the amount of donated blood to the total amount of blood in the body?



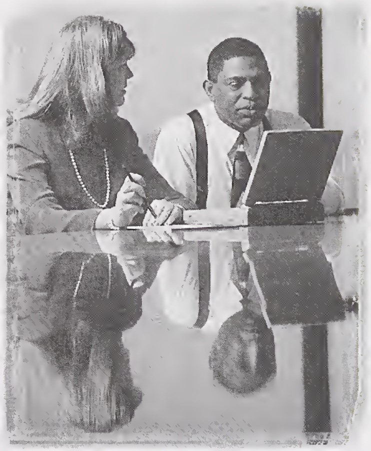
Check your answers by turning to the Appendix, page 175.

Proportional Ratios

If you think of a ratio as a rule describing a relationship, it is possible to write many proportional or equivalent ratios.

Example

Suzanne and Peter invest in a business. Suzanne invests \$5 for every \$4 that Peter invests. Express this relationship as two other equivalent ratios.



Solution

An infinite number of ratios can be written using the rule \$5 to \$4 or 5:4.

- Suzanne could invest \$500, and Peter could invest \$400.

$$\frac{5}{4} \underset{\times 100}{\overset{\times 100}{=}} \frac{500}{400} \qquad \frac{\text{Suzanne s investment (\$)}}{\text{Peter s investment (\$)}}$$

So, the ratio of Suzanne's investment to Peter's investment would be 500 to 400.

- Suzanne could invest \$10 000, and Peter could invest \$8000.

$$\frac{5}{4} \underset{\times 2000}{\overset{\times 2000}{=}} \frac{10\,000}{8000} \qquad \frac{\text{Suzanne s investment (\$)}}{\text{Peter s investment (\$)}}$$

So, the ratio of Suzanne's investment to Peter's investment would be 10 000 to 8000.

Note: In the preceding example, the ratios are proportional ratios. The equations $\frac{5}{4} = \frac{500}{400}$ and $\frac{5}{4} = \frac{10\,000}{8000}$ are proportions.

In questions 9 and 10, write statements and express the ratios in fraction form.

9. When you make orange juice from cans of concentrate, the rule is 1 can of concentrate to 3 cans of water. Write three proportional ratios to describe the situation.

10. It has been recommended that the optimum number of students to the number of teachers is 20 to 1. Write equivalent ratios to describe this situation.

Check your answers by turning to the Appendix, page 176.

Now Try This

Use a problem-solving strategy to answer the following question.

11. Beth has only a 5-L pail and a 3-L pail. Neither of the pails has any markings on it.

- **a.** Explain how Beth can measure exactly 2 L.
- **b.** Explain how Beth can measure exactly 1 L.
- **c.** Explain how Beth can measure exactly 4 L.

Check your answers by turning to the Appendix, page 176.

Looking Back

In this activity, you wrote ratios to compare two quantities measured in the same unit or in units that can be converted to the same unit.

12. In your journal, explain the meaning of the following directions:

> "It is easy to cook rice. You just have to remember it is 2 water to 1 rice."

Check your answers by turning to the Appendix, page 176.

Activity 2: Rates

Comparing the Number of People, Animals, or Things with a Quantity

The most densely populated region in the world is Macau, on the southern coast of China. In 1998, Macau had an estimated population of 430 000 in an area of $21\ \text{km}^2$.

A rate can be used to compare the number of people in Macau to the area of Macau.

A rate is a comparison that describes a unit relationship as well as a number relationship.

Rates can be written in different ways.

Example

When you say that a person can type 700 words in 9 min, you are comparing the number of words typed to the amount of time required. Write the typing speed as a rate.

Solution

Method 1: Using the Word *per*

The typist typed 700 words per 9 min.

Method 2: Using a Colon

The typist typed 700 words : 9 min .

Each of the forms in Methods 2 to 4 are read as "700 words per 9 min."

Method 3: Using Fraction Form

The typist typed $\frac{700\text{ words}}{9\text{ min}}$.

Method 4: Using a Slash

The typist typed 700 words/9 min.

Note: Because the number of words typed is being compared to the amount of time, the category *words* and the unit *minute* are included as part of the rate.

Answer the following question involving rate.

1. George paid $3.99 for 12 doughnuts. Write the rate of cost in each of the following forms.

   a. using the word *per*
   b. using a colon
   c. using fraction form
   d. using a slash

Check your answers by turning to the Appendix, page 177.

Comparing Quantities Expressed in Different Units

The world's most accurate mechanical clock is at the Copenhagen City Hall in Denmark. The clock is accurate to 0.5 seconds in 300 years. This means the clock will lose or gain no more than 0.5 seconds in 300 years.

A rate can be used to compare the number of seconds that could be lost or gained to the number of years.

Remember, rates can be written in different ways.

Example

When you say that a bus from Copenhagen, Denmark, to Berlin, Germany, travelled a distance of 360 km in 4 h, you are comparing the amount of distance travelled to the amount of time required. Write the speed of the bus as a rate.

Solution

Method 1: Using the Word *per*

The bus travelled at 360 km per 4 h.

Method 2: Using a Colon

The bus travelled at $360\ \text{km} : 4\ \text{h}$.

Each of the forms in Methods 2 to 4 are read as "360 km per 4 h."

Method 3: Using Fraction Form

The bus travelled at $\frac{360\ \text{km}}{4\ \text{h}}$.

Method 4: Using a Slash

The bus travelled at 360 km/4 h.

Note: Because the quantities being compared have different units, the units are included as part of the rate.

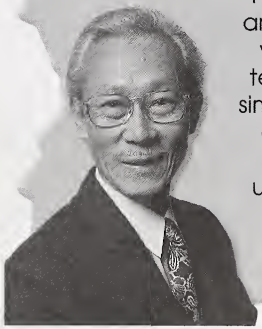
2. A rubber band stretched 1 cm when a 25-g mass was hung on it. Write the rate of stretch in each of the following forms:

   a. using the word *per*
   b. using a colon
   c. using fraction form
   d. using a slash

3. An advertisement stated that 4 L of paint would cover $25\ \text{m}^2$. Write the rate of coverage in each of the following forms:

   a. using the word *per*
   b. using a colon
   c. using fraction form
   d. using a slash

Check your answers by turning to the Appendix, page 177.



Previously, you found that ratios are simplified by expressing them with the lowest whole-number terms possible. Rates are usually simplified by expressing them with a second term of 1. Often the second term of 1 is merely understood and it is not written.

Simplifying Rates

Example

A department store is having a shoe sale. To increase sales, it advertises the cost of $10 for 8 pairs of socks as a bonus. Express the rate of cost to the number of socks in simplest form. The cost of each pair of socks can be written as a rate in simplest form.

Solution

$$\frac{10}{8} \qquad \frac{\text{cost (\$)}}{\text{number of pairs}}$$

Simplify the rate to show the cost per 1 pair of socks.

$$\frac{10}{8} \overset{\div 8}{=} \frac{1.25}{1}$$

The cost is $1.25 per pair of socks.

Example

A June rainstorm produced 81.6 mm of rain in 48 h. Express this relationship as a rate in simplest form.

Solution

$$\frac{81.6 \text{ mm}}{48 \text{ h}} \qquad \frac{\text{amount of rain (mm)}}{\text{amount of time (h)}}$$

Simplify the rate to show the rainfall in 1 h.

$$\frac{81.6}{48} \overset{\div 48}{=} \frac{1.7}{1}$$

The rate of rainfall was 1.7 mm/h.

4. Write each of the following statements as rates in simplest form.

a. Bobby can type 420 words in 4 min.
b. Dorothy's heart beats 300 times in 4 min.
c. A rubber band stretches 2 cm when supporting a mass of 100 g.
d. Ahmad exchanged CDN$215 for £100 (British pounds).

Check your answers by turning to the Appendix, page 177.

Proportional Rates

If you think of a rate as a rule, it is possible to write many proportional or equivalent rates.

Example

A supermarket sells pork chops for the price of $4.30 per kg. Write a number of proportional rates illustrating this cost.

Solution

An infinite number of rates can be written using this rule.

- The cost could be $8.60 per 2 kg.

$$\frac{4.30}{1} \overset{\times 2}{=} \frac{8.60}{2} \qquad \frac{\text{cost (\$)}}{\text{mass (kg)}}$$

- The cost could be $12.90 per 3 kg.

$$\frac{4.30}{1} \overset{\times 3}{=} \frac{12.90}{3} \qquad \frac{\text{cost (\$)}}{\text{mass (kg)}}$$

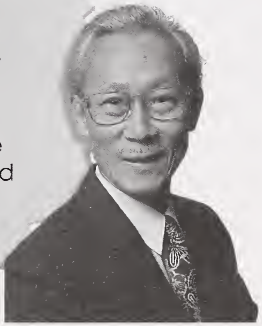
Note: The ratios in the example are proportional. The equations $\frac{4.30}{1} = \frac{8.60}{2}$ and $\frac{4.30}{1} = \frac{12.90}{3}$ are proportions.

Complete the following question involving proportional rates.

5. Arlene can swim 2.5 m per second. Write three proportional rates to describe her rate of speed swimming.

Check your answer by turning to the Appendix, page 178.

What lets you distinguish between a ratio and a rate?

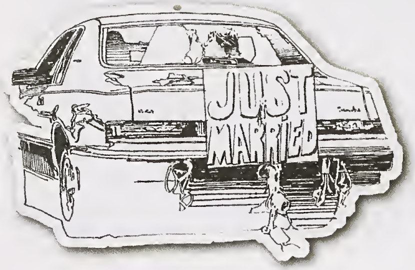


Usually the comparison of the number of people, animals, or things in different groups is expressed as a ratio. However, when the comparison uses the word *per*, it is sometimes referred to as a rate.

Birth rates, marriage rates, and divorce rates are expressed as the number of births, marriages, or divorces per 1000 population.

Example

There were 156 691 marriages in Canada in 1996. If the population in 1996 was about 29 672 000, what was the marriage rate? (Remember that such rates are usually expressed per 1000 population.)



Solution

$$\frac{156\,691}{29\,672\,000} \overset{\div 29\,672}{=} \frac{5.28}{1000}$$

$$\frac{\text{number of marriages}}{\text{total number of people}}$$

In 1996, the marriage rate in Canada was about 5.28 marriages per 1000 population.

6. There were 71 528 divorces in Canada in 1996. If the population in 1996 was about 29 672 000, what was the divorce rate?

Check your answer by turning to the Appendix, page 178.

Comparing Ratios and Rates

At one store, bananas cost $7.35 for 5 kg. In another store, bananas cost $4.41 for 3 kg. Which store is offering the better deal?

To compare the prices, write the rates in simplest form.

Store 1

$$\frac{7.35}{5} = \frac{1.47}{1}$$

Store 2

$$\frac{4.41}{3} = \frac{1.47}{1}$$

$$\frac{\text{price (\$)}}{\text{mass (kg)}}$$

Because both rates in simplest form are equal to $1.47/kg, the stores are offering the same deal.

Comparing ratios and rates is a useful skill.

Example

Julia likes to paint. Will the ratio of 15 mL of blue to 10 mL of yellow paint produce the same colour of green as the ratio of 21 mL of blue paint to 14 mL of yellow paint?

Solution

Step 1: Write the ratios in simplest form.

Green 1

$$\frac{15}{10} = \frac{3}{2}$$

Green 2

$$\frac{21}{14} = \frac{3}{2}$$

$$\frac{\text{amount of blue paint (mL)}}{\text{amount of yellow paint (mL)}}$$

Step 2: Compare the ratios.

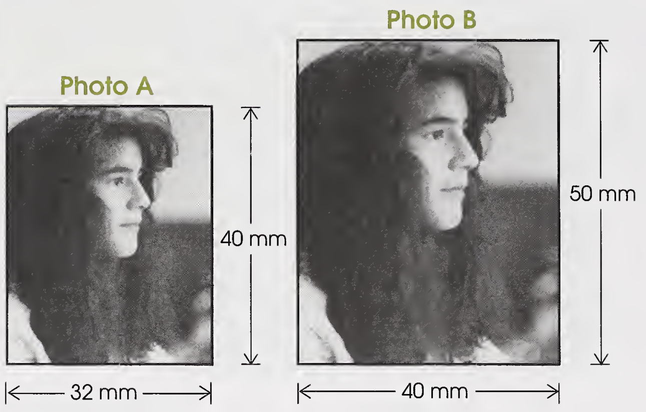
Because both ratios in simplest form are $\frac{3}{2}$, they are proportional.

So, the same green colour will be produced.

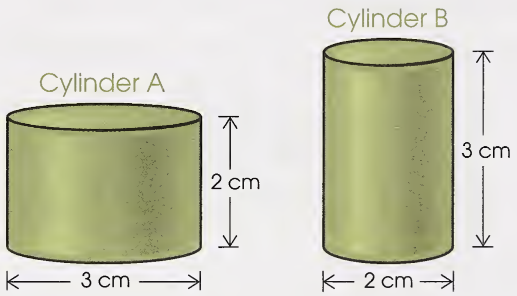
Note: Be careful of the order of terms when comparing ratios or rates.

Complete questions 7 to 12.

7. Compare the ratios of the lengths to the widths of the following photographs. Are they equivalent?



8. Compare the ratios of the heights to the diameters of the following cylinders. Are they equivalent?



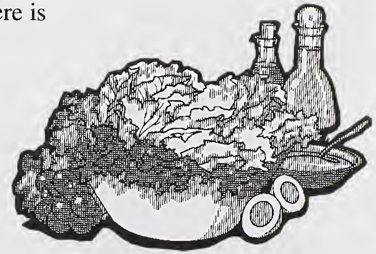
9. Dino travelled 830 km in 9 h. Frank travelled 640 km in 7 h. Who travelled at a faster rate of speed?

10. The following chart gives the quantities of rice required to serve 2, 4, and 6 people.

	Servings		
	2	4	6
Rice (mL)	125	175	250
Water (mL)	300	375	500
Butter (mL)	5	10	15
Salt (mL)	2	5	7

Is the ratio of the amount of rice to the amount of water proportional for 2, 4, and 6 servings? Answer **yes** or **no** and explain why.

11. In a certain kind of salad dressing, there is a ratio of 2 parts oil to 1 part vinegar. Could the amounts in each of the following situations be the amount of oil and vinegar in the salad dressing? Answer **yes** or **no** and explain why.



a. 60 mL of oil and 30 mL of vinegar
b. 30 mL of oil and 60 mL of vinegar
c. 90 mL of oil and 45 mL of vinegar
d. 45 mL of oil and 90 mL of vinegar

12. The following chart gives the cost for 2 kg, 4 kg, and 6 kg of ground beef. Are the rates proportional? Answer **yes** or **no** and explain why.

Cost ($)	8.60	17.20	25.80
Mass (kg)	2	4	6

Check your answers by turning to the Appendix, page 178.

Looking Back

In this activity, you worked with rates to compare the number of people, animals, or things in a group to a quantity. You wrote rates to compare quantities expressed in two different units.

13. In your journal, write at least three rates you use or see daily.

14. Earlier in this activity, you saw that birth rates, marriage rates, and divorce rates are expressed per 1000 population. Why do you think 1000 was chosen?

15. Check back to the introduction of Activity 2. What is the population per square kilometre in Macau?

Check your answers by turning to the Appendix, page 179.

Activity 3: Percents

Did you know that the Inuit have many ways to describe snow?

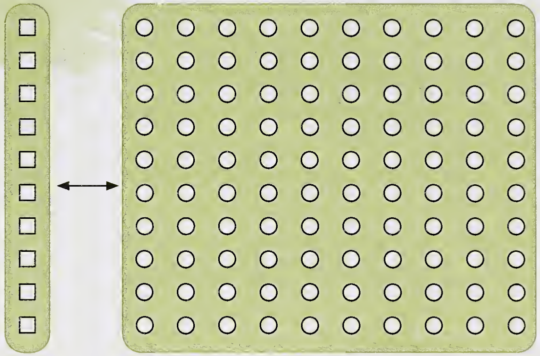
Just as every language has synonyms to describe objects or ideas, mathematics has many equivalent ways to describe relationships.

In this activity, you will investigate ratios and how they can be used to describe relationships. You will write each ratio as a fraction, as a decimal number, and as a percent.

A percent is a special ratio with 100 as the second term. Percent means per hundred.

Example

There are two groups of shapes in the given diagram: a group of 10 squares and a group of 100 circles.



The ratio of the number of squares to the number of circles is 10 to 100. Express this ratio as a fraction, as a decimal number, and as a percent.

Solution

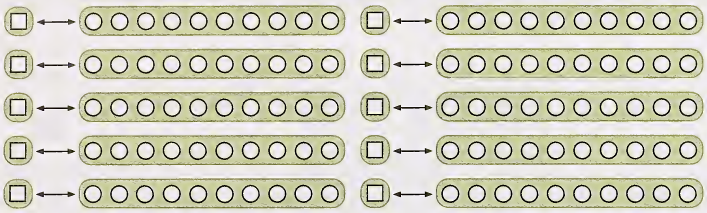
Method 1: As a Fraction

$$\frac{10}{100} = \frac{1}{10}$$

$$\frac{\text{number of squares}}{\text{number of circles}}$$

The number of squares in the diagram is $\frac{1}{10}$ of the number of circles.

Regrouping the squares and circles may help you visualize the fraction form.



There is 1 square for every 10 circles in the diagram.

Method 2: As a Decimal Number

$\frac{10}{100} = 0.1$ ← Divide the numerator by the denominator.

$\frac{\text{number of squares}}{\text{number of circles}}$

The number of squares in the diagram is 0.1 of the number of circles.

Method 3: As a Percent

$\frac{10}{100} = 10\%$

$\frac{\text{number of squares}}{\text{number of circles}}$

$\frac{10}{100}$ means 10 per hundred or 10 percent.

The number of squares in the diagram is 10% of the number of circles.

1. A bowl of a raisin and bran cereal contains 20 raisins to 100 bran flakes. Express this relationship in the following ways:

   a. as a fraction in simplest form
   b. as a decimal number
   c. as a percent

2. The ratio of the length of a grasshopper's body to the distance the grasshopper can jump is 5 to 100. Express this relationship in the following ways:

   a. as a fraction in simplest form
   b. as a decimal number
   c. as a percent

Check your answers by turning to the Appendix, page 179.

Remember: When you are dividing a quantity into parts, the percentages the parts represent must total 100%.

Example

You can purchase bread that is 60% whole wheat. What percent of the flour is white?

Solution

The flour used to make 60% whole-wheat bread is a blend of white flour and whole-wheat flour; 60 parts out of 100 parts of flour are whole-wheat flour.

If all the flour was whole-wheat flour, it would be 100% whole wheat.

So, $100\% - 60\% = 40\%$ of the flour is white.

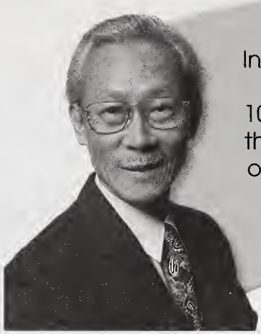
Note: This means 40 out of 100 parts of flour is white.

3. Ramon won 30% of the ski races he entered. What percent of the races did he lose?

4. In a group of children, 5% are left-handed. What percent are right-handed?

5. In a certain group, 20% wear glasses. What percent do not wear glasses?

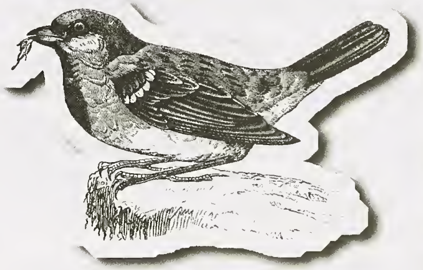
Check your answers by turning to the Appendix, page 180.



In questions 1 and 2, each of the ratios have a second term of 100. It is possible to express ratios that do not have a second term of 100 as a percent by finding a proportional ratio.

Example

The ratio of the time for a sparrow egg to hatch to the time for a penguin egg to hatch is 1 to 5. Express this ratio as a fraction, a decimal number, and a percent.



Solution

Method 1: As a Fraction

$$\frac{1}{5} \qquad \frac{\text{time for hatching a sparrow egg}}{\text{time for hatching a penguin egg}}$$

The time for a sparrow egg to hatch is $\frac{1}{5}$ of the time for a penguin egg to hatch.

Method 2: As a Decimal Number

$$\frac{1}{5} = \frac{2}{10} = 0.2 \qquad \frac{\text{time for hatching a sparrow egg}}{\text{time for hatching a penguin egg}}$$

The time for a sparrow egg to hatch is 0.2 of the time for a penguin egg to hatch.

Method 3: As a Percent

$$\frac{1}{5} \overset{\times 20}{\underset{\times 20}{=}} \frac{20}{100} \qquad \frac{\text{time for hatching a sparrow egg}}{\text{time for hatching a penguin egg}}$$

The time for a sparrow egg to hatch is 20% of the time for a penguin egg to hatch.

6. In a computer class, the ratio of the number of students present to the total number of students is 3 to 4. Express this ratio in the following ways:

   a. as a fraction in simplest form
   b. as a decimal number
   c. as a percent

7. The ratio of the length of a nail to the length of a pencil is 3 to 5. Express the ratio in the following ways:

   a. as a fraction in simplest form
   b. as a decimal number
   c. as a percent

8. About $\frac{5}{8}$ of the pets in Vasili's neighbourhood are dogs. Express this fraction as a percent.

Check your answers by turning to the Appendix, page 180.

Example

Container C has 750 mL of popcorn. Container D has 375 mL of popcorn. Therefore, the ratio of the amount of popcorn in Container C to the amount of popcorn in Container D is 750 to 375, or 2 to 1. Express the ratio as a whole number and as a percent.

Solution

Method 1: As a Whole Number

$$\frac{750}{375} = \frac{2}{1} \qquad \frac{\text{amount of popcorn in C (mL)}}{\text{amount of popcorn in D (mL)}}$$
$$= 2$$

The amount of popcorn in Container C is 2 times the amount of popcorn in Container D.

Method 2: As a Percent

$$\frac{750}{375} = \frac{2}{1} \qquad \frac{\text{amount of popcorn in C (mL)}}{\text{amount of popcorn in D (mL)}}$$
$$= \frac{200}{100}$$
$$= 200\%$$

The amount of popcorn in Container C is 200% of the amount of popcorn in Container D.

Example

The length of a person's small intestine is 4 times the length of a person's large intestine. Express this whole number as a percent.

Solution

$$4 = \frac{4}{1}$$
$$= \frac{400}{100}$$
$$= 400\%$$

The length of a person's small intestine is 400% of the length of the large intestine.

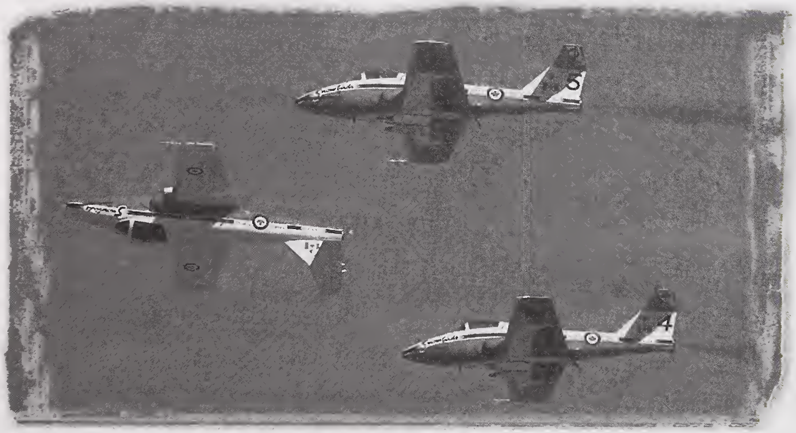
9. A cranapple drink is made with 375 mL of cranberry juice and 375 mL of apple juice. What is the ratio of the amount of cranberry juice to the amount of apple juice?

   a. Express the ratio in lowest terms.
   b. Express the ratio as a whole number.
   c. Express the ratio as a percent.

10. On Earth, a high jumper can jump 2 m. Scientists estimate that on the Moon, the same high jumper could jump 12 m. What is the ratio of the height of the jump on the Moon to the height of the jump on Earth? Express this ratio in the following ways:

    a. as a ratio using a colon
    b. as a whole number
    c. as a percent

11. Some jet planes travel at a speed that is about 6 times the flying speed of the spine-tail swift. Express this ratio as a percent.



12. A supersonic jet can fly at a speed that is 2 times the speed of sound. Express this ratio as a percent.

13. The mass of an ostrich egg is about 2700 times the mass of a hummingbird egg. Express this ratio as a percent.

Check your answers by turning to the Appendix, page 181.

Example

Joan borrowed \$20 from her mother and repaid her \$22. So, the ratio of the amount of money Joan repaid her mother to the amount of money she borrowed is 22 to 20. Express this ratio as a mixed number, a decimal number, and a percent.

Solution

Method 1: As a Mixed Number

$$\frac{22}{20} = \frac{11}{10}$$
$$= 1\frac{1}{10}$$

$\dfrac{\text{amount repaid (\$)}}{\text{amount borrowed (\$)}}$

Joan repaid $1\frac{1}{10}$ of the money she borrowed.

Method 2: As a Decimal Number

$$\frac{22}{20}=\frac{11}{10}$$
$$=1.1$$

$\dfrac{\text{amount repaid (\$)}}{\text{amount borrowed (\$)}}$

Joan repaid 1.1 times the money she borrowed.

Method 3: As a Percent

$$\frac{22}{20}=\frac{11}{10}$$
$$=\frac{110}{100}$$
$$=110\%$$

$\dfrac{\text{amount repaid (\$)}}{\text{amount borrowed (\$)}}$

Joan repaid 110% of the money she borrowed.

14. There were 24 canoes and 15 sailboats on the lake. What is the ratio of the number of canoes to the number of sailboats? Express this ratio in each of the following ways:

a. as a ratio with the word *to*
b. as a mixed number
c. as a decimal number
d. as a percent

Check your answers by turning to the Appendix, page 182.

Expressing Decimal Numbers as Percents

Sometimes you may wish to change a ratio expressed as a decimal number to its percent form.

Example

The 2000 Canadian $200 coin depicting an Inuit mother and child is an alloy of gold and silver; 0.9167 of the coin is gold. Express this decimal number as a percent.

[1]

Solution

Step 1: Write the decimal number as an equivalent ratio with a second term of 100.

$$0.9167=\frac{91.67}{100}$$

Step 2: Write the equivalent ratio as a percent.

$$\frac{91.67}{100}=91.67\%$$

So, 91.67% of the 2000 gold coin is gold.

[1] Coin designs courtesy of the Royal Canadian Mint/Image de la pièce, courtoisie de la Monnaie royale canadienne.

Example

The speed at which the canvasback duck can fly is 2.3 times the speed at which an ostrich can run. Express this ratio as a percent.

Solution

$$\begin{aligned}2.3 &= \frac{2.3}{1}\\ &= \frac{230}{100}\\ &= 230\%\end{aligned}$$

The speed at which a canvasback duck can fly is 230% of the speed at which an ostrich can run.

15. Advanced tickets to a concert are sold at 0.75 of the cost at the door. Write this decimal number as a percent.

16. During the 1914 baseball season, Ty Cobb had a batting average of 0.390. This means he hit safely 0.390 of the times he was up to bat. Write the decimal number as a percent.

17. The mass of milk is 1.56 times the mass of gasoline. Express this ratio as a percent.

Check your answers by turning to the Appendix, page 183.

Expressing Percents as Decimal Numbers and Fractions

It may be useful to be able to express percents as decimal numbers and fractions.

Example

An alloy of copper, zinc, and lead is used to make the parts of a watch. Copper is 64% of the alloy. Express this ratio as a decimal number and a fraction.

Solution

Method 1: As a Decimal Number

$$\begin{aligned}64\% &= \frac{64}{100} &&\longleftarrow \text{Write the percent as a ratio with a second term of 100.}\\ &= 0.64 &&\longleftarrow \text{Write the ratio as a decimal number.}\end{aligned}$$

Copper is 0.64 of the alloy.

Method 2: As a Fraction

$$\begin{aligned}64\% &= \frac{64}{100} &&\longleftarrow \text{Write the percent as a ratio with a second term of 100.}\\ &= \frac{16}{25} &&\longleftarrow \text{Write the ratio as a fraction in simplest form.}\end{aligned}$$

Copper is $\frac{16}{25}$ of the alloy.

Example

The Smiths sold their house for 125% of what they paid for it four years earlier. Express this percent as a mixed number and a decimal number.

Solution

Method 1: As a Mixed Number

$$125\% = \frac{125}{100}$$
$$= 1\frac{25}{100}$$
$$= 1\frac{1}{4}$$

The Smiths sold their house for $1\frac{1}{4}$ times the purchase price.

Method 2: As a Decimal Number

$$125\% = \frac{125}{100}$$
$$= 1.25$$

The Smiths sold their house for 1.25 times the purchase price.

18. The elementary school students sold chocolate bars to raise money. They kept 20% of the sales. Express this percent in each of the following forms:

a. as a decimal number
b. as a fraction in simplest form

19. In a basketball game, Darryl made 35% of the shots he attempted. Express this percent in each of the following forms:

a. as a decimal number
b. as a fraction in simplest form

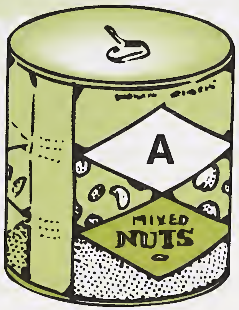
Check your answers by turning to the Appendix, page 183.

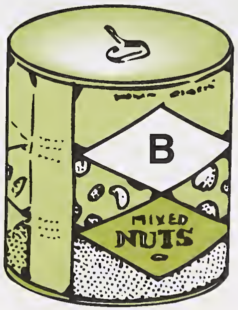
Using Percents to Compare

Another way to compare ratios is to write them in percent forms.

Example

In Can A, the ratio of the number of cashews to the total number of mixed nuts is 1 to 5. In Can B, the ratio of the number of cashews to the total number of mixed nuts is 3 to 10. Which can has the higher ratio of cashews to mixed nuts?





Solution

Step 1: Write the ratios as percents.

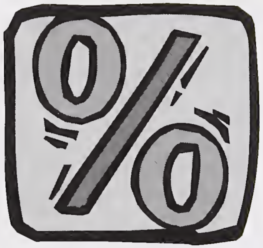
Can A	Can B
$\frac{1}{5} = \frac{20}{100}$	$\frac{3}{10} = \frac{30}{100}$
$= 20\%$	$= 30\%$

$$\frac{\text{number of cashews}}{\text{total number of mixed nuts}}$$

Step 2: Compare the percents.

$30\% > 20\%$

Can B has the higher ratio of cashews to mixed nuts.



20. Kara scored 17 out of 25 on the first test and 14 out of 20 on the second test. On which test did she do better?

21. Haden and Steven play wheelchair basketball. Haden sank 7 out of 10 shots. Steven sank 13 out of 20 shots. Who is the more accurate shooter?

Check your answers by turning to the Appendix, page 184.

Using Decimal Forms to Compare

Another way to compare ratios is to write them in their decimal forms.

Example

Luke hits safely 3 out of 8 times at bat. Jerritt hits safely 4 out of 9 times at bat. Who has the greater ratio of hits to times at bat?

Solution

Step 1: Write the ratios in decimal form. Round to the nearest thousandth.

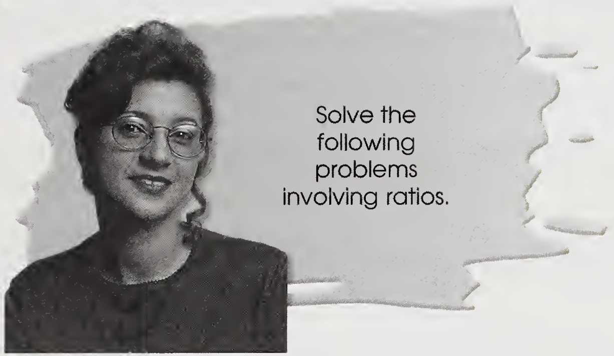
Luke	Jerritt
$\frac{3}{8} = 3 \div 8$	$\frac{4}{9} = 4 \div 9$
$= 0.375$	$\doteq 0.444$

$$\frac{\text{number of hits}}{\text{number of times at bat}}$$

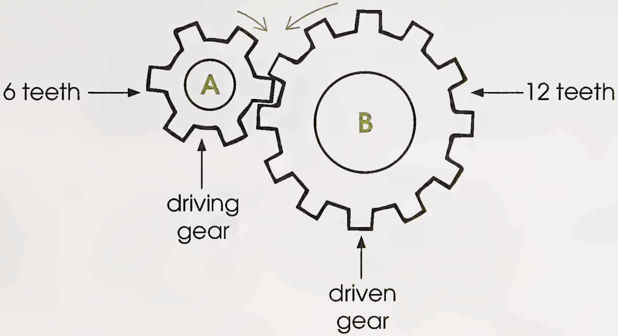
Step 2: Compare the decimal forms.

$0.444 > 0.375$

Jerritt has the greater ratio of hits to times at bat.

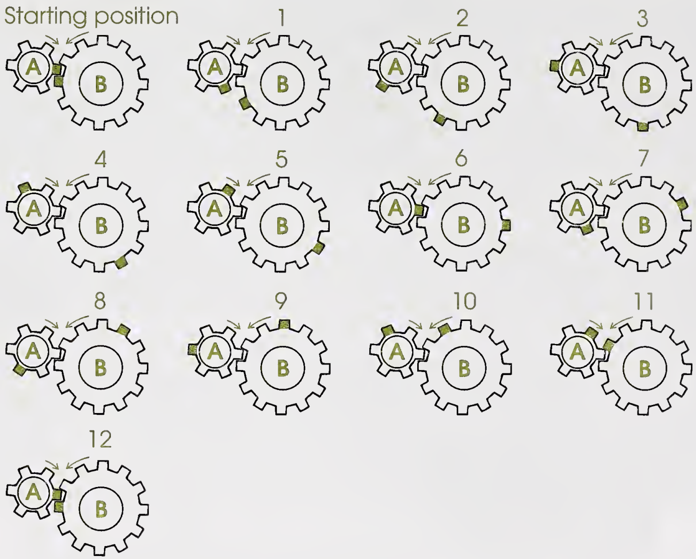


22. **a.** Examine the following diagram of a gear train. Notice that there are two gears. Gear A is the driving gear, and gear B is the driven gear.



What is the **gear ratio**? **Hint:** The gear ratio is the ratio of the number of teeth on the driving gear to the number of teeth on the driven gear.

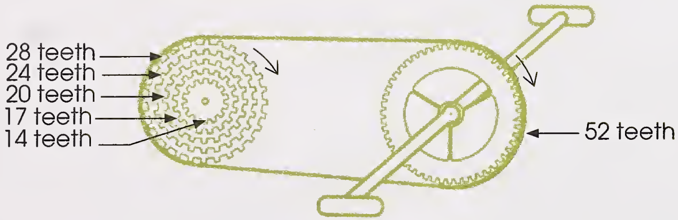
b. Look at the following diagram. Notice how the teeth of the two gears mesh. As Gear A moves, it drives Gear B. Also, notice the direction each gear is moving. Gear A moves clockwise, and Gear B moves counterclockwise.



How many revolutions (complete turns) do Gear A and Gear B each make? **Hint:** Examine the position of the coloured teeth on each of the gears in the diagram.

c. What is the **turn ratio** of the gears in question 22.b.? **Hint:** The turn ratio is the ratio of the number of turns of the driving gear to the number of turns of the driven gear.

23. Examine the following simplified diagram of the gears on a five-speed bicycle. There is one gear at the pedals; it is the driving gear. There are five gears on the back wheel; they are the driven gears. The gears are attached by a chain and can be connected in different positions. Notice that the front and back gears move in the same direction.

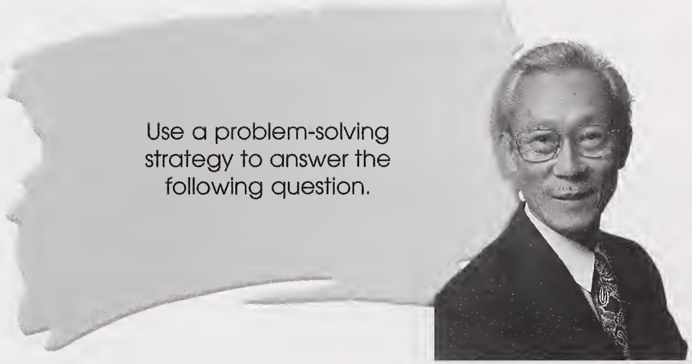


Complete a chart like the one that follows by finding the gear ratios and turn ratios in decimal form. Round to the nearest thousandth.

Gear	Number of Teeth on Front Gear	Number of Teeth on Back Gear	Gear Ratio	Turn Ratio
1st	52	28		
2nd	52	24		
3rd	52	20		
4th	52	17		
5th	52	14		

Check your answers by turning to the Appendix, page 184.

Now Try This



24. One wall in a museum is 10 m long. Picture hooks are to be placed every metre along the wall. How many hooks will be needed?

Check your answer by turning to the Appendix, page 184.

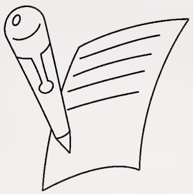
Looking Back

In this activity, you wrote ratios as fractions, as decimal numbers, and as percents. You expressed decimal numbers and fractions as percents. You changed percents to decimal numbers and fractions. You also compared ratios and rates.

25. In your journal, complete the table and write a short explanation describing how you converted these numbers to the different forms.

Fraction	Decimal	Percent
$\frac{9}{20}$		
	0.15	
		55%

Check your answer by turning to the Appendix, page 185.



Activity 4: Solving Ratio, Rate, and Percent Problems

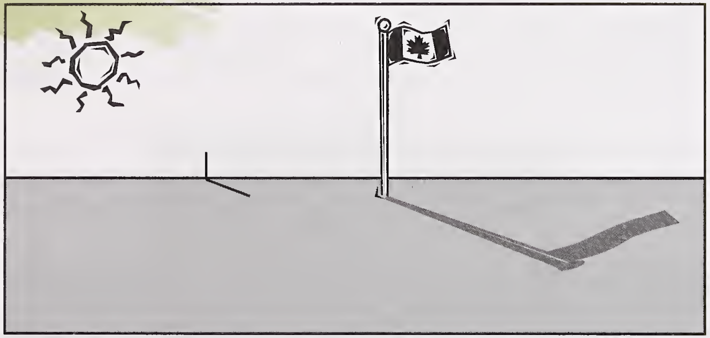
Natalie went shopping to take advantage of an end-of-season sale. All of the items she purchased were 75% off the regular price. If she spent $175, how much did she save?

One of the main purposes of mathematics is to solve problems. In this activity, you will use reasoning and your knowledge of proportions to solve ratio, rate, and percent problems.

Proportional reasoning allows you to solve ratio problems like the following.

Example

Malachi measures the shadow of a metre stick and the shadow of a flagpole at the same time of day. If the shadow of the metre stick is 3 m long and the shadow of the flagpole is 15 m long, how tall is the flagpole?



Solution

Step 1: To solve the problem, write a proportion.

$$\frac{1}{3} = \frac{\square}{15}$$

flagpole

metre stick

$$\frac{\text{height of object (m)}}{\text{length of shadow (m)}}$$

Step 2: Find the missing term.

$$\frac{1}{3} \overset{\times 5}{=} \frac{5}{15}$$

$$\frac{\text{height of object (m)}}{\text{length of shadow (m)}}$$

The flagpole is 5 m tall.

Complete the following problems using proportions.

1. A cookie recipe calls for 2 parts flour to 1 part sugar. If Tom uses 350 mL of flour, how much sugar does he need to make these cookies?

2. In a certain restaurant, the ratio of the number of cooks to the number of waiters and waitresses is 3:5. If there are 9 cooks, how many waiters and waitresses are there?

3. The ratio of canoes to sailboats on Thunder Lake is 7 to 2. If there are 21 canoes, how many sailboats are there?

4. The ratio of the number of bones in your head to the total number of bones in your body is about 1 to 7. If your body contains 206 bones, what is the approximate number of bones in your head?

Check your answers by turning to the Appendix, page 185.

Proportional reasoning allows you to solve ratio problems like the following.

Example

The ratio of the number of domestic cars to the number of foreign cars in a certain city is about 9 to 5. Estimate the number of foreign cars you could expect to find in a parking lot holding 280 cars.

Solution

If the ratio of the number of domestic cars to the number of foreign cars in a city is 9 to 5, the ratio of the number of foreign cars to the total number of cars in the city is 5 to 14.

Step 1: To solve the problem, write a proportion.

$$\frac{5}{14} = \frac{\boxed{}}{280} \qquad \frac{\text{number of foreign cars}}{\text{total number of cars}}$$

Step 2: Find the missing term.

$$\frac{5}{14} \overset{\times 20}{=} \frac{100}{280} \qquad \frac{\text{number of foreign cars}}{\text{total number of cars}}$$

You could expect to find about 100 foreign cars in the parking lot.

5. Sterling silver is an alloy of silver and copper in the ratio of 37 to 3. How many grams of silver are there in a sterling silver cup with a total mass of 400 g?

Check your answer by turning to the Appendix, page 186.

Proportional reasoning also allows you to solve rate problems like the following.

Example

Jessica is watching a storm. She sees lightning and 8 s later she hears thunder. How far away is the storm? **Hint:** Sound travels at the rate of 344 m/s.

Solution

Step 1: To solve the problem, write a proportion.

$$\frac{344}{1} = \frac{\boxed{}}{8} \qquad \frac{\text{distance (m)}}{\text{time (s)}}$$

Step 2: Find the missing term.

$$\frac{344}{1} \overset{\times 8}{\underset{\times 8}{=}} \frac{\boxed{2752}}{8} \qquad \frac{\text{distance (m)}}{\text{time (s)}}$$

The storm is 2752 m away.

Example

Arthur is training for a 10-km race. He jogs 6 km daily and it takes him an average of 30 min to jog this distance. At this rate, how long will it take him to finish the 10-km race?

Solution

Step 1: Find Arthur's average rate of jogging speed.

$$\frac{6}{30} = \frac{0.2}{1} \qquad \frac{\text{distance (km)}}{\text{time (min)}}$$

Arthur jogs at a rate of 0.2 km/min.

Step 2: To solve the problem, write a proportion.

$$\frac{0.2}{1} = \frac{10}{\boxed{}} \qquad \frac{\text{distance (km)}}{\text{time (min)}}$$

Step 3: Find the missing term.

$$\frac{0.2}{1} \overset{\times 50}{\underset{\times 50}{=}} \frac{10}{\boxed{50}} \qquad \frac{\text{distance (km)}}{\text{time (min)}}$$

It will take Arthur 50 min to finish the 10-km race at this rate.

Solve the following rate problems.

6. If Arlene swims at a rate of 2.5 m/s, how far can she swim in 50 s?

7. René, who works a 40-h week, is paid $450 per week. At the same hourly wage, how much will he receive if he works a 36-h week?

8. Chester travels 25 km in 2.5 h on his bike. At that rate, how long will it take him to travel 50 km?

Check your answers by turning to the Appendix, page 186.

Proportional reasoning also allows you to solve currency problems.

Example

The following table shows the average cost of buying foreign currency in Canadian dollars in September 2000.

Currency	Exchange Rate
U.S. dollar	1.472
Euro	1.314
British Pound	2.136
Japanese Yen	0.014
German Deutsche Mark	0.672
French Franc	0.200

Using the table, you can buy U.S.$1 for CDN$1.47. How much does it cost to buy U.S.$100?

Solution

Step 1: Find the exchange rate.

$$\frac{1.47}{1} \qquad \frac{\text{amount (CDN \$)}}{\text{amount (U.S. \$)}}$$

Step 2: To solve the problem, write a proportion. Find the missing term.

$$\frac{1.47}{1} \overset{\times 100}{\underset{\times 100}{=}} \frac{147}{100} \qquad \frac{\text{amount (CDN \$)}}{\text{amount (U.S. \$)}}$$

It costs CDN$147 to buy U.S.$100.

9. Using the exchange rates from the table in the example, you can purchase 1000 Japanese yen for $14 Canadian. How many Japanese yen can be bought for $70 Canadian?

Check your answer by turning to the Appendix, page 186.

Solving Percent Problems

Proportional reasoning also allows you to solve percent problems.

Example

The volleyball team sold $375 worth of chocolate bars to go towards new uniforms. If the team kept 20% of the sales, how much did they keep?

Solution

Step 1: Write the percent as a ratio in simplest form.

$$20\% = \frac{20}{100}$$
$$= \frac{1}{5}$$

Step 2: To solve the problem, write a proportion and find the missing term.

$$\frac{1}{5} = \frac{\square}{375} \qquad \frac{1}{5} \overset{\times 75}{=} \frac{75}{375}$$

$$\frac{\text{amount kept (\$)}}{\text{amount sold (\$)}}$$

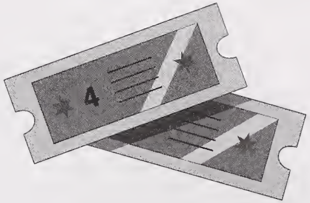
The team kept $75.

Example

Advanced tickets to a rock concert are sold for $20 each. This is 80% of the price of the tickets at the door. How much will a ticket cost at the door?

Solution

Step 1: Write the percent as a ratio in simplest form.



$$80\% = \frac{80}{100}$$
$$= \frac{4}{5}$$

Step 2: To solve the problem, write a proportion and find the missing term.

$$\frac{4}{5} = \frac{20}{\square} \qquad \frac{4}{5} \overset{\times 5}{=} \frac{20}{25}$$

$$\frac{\text{price of advance tickets (\$)}}{\text{cost of tickets at door (\$)}}$$

The price of a ticket at the door is $25.

Solve the following problems involving percents.

10. A package of cheese has 23% milk fat. If the package holds 200 g of cheese, how much milk fat is in the package?

11. The Allans left their waiter a tip equal to 15% of the food bill. If the tip was $9, what was the food bill?

12. This year the profits of a local business were 130% of the profits last year. If the profits last year were $5200, what were the profits this year?

13. A survey found that at a certain school, 120 students (or about 80% of the students) held part-time jobs. How many students attend the school?

Check your answers by turning to the Appendix, page 187.

Now Try This

Use a problem-solving strategy to answer the following questions.

14. A farmer had a herd of cattle that he sold to four buyers. He sold 50% of the cattle to the first buyer, 25% of the cattle to the second buyer, 20% of the cattle to the third buyer, and 7 cattle to the fourth buyer. How many cattle did the farmer have?

15. Zoe's mark on a test was 80%. If there were 25 questions of equal value on the test, what was the ratio of the number of correct responses to the number of incorrect responses on Zoe's test?

16. Geoff can run 5 m/s. Simon can run 7.5 m/s. If Geoff had a head start of 10 m, would he win a 100-m race?

17. Suki's record for the 100-m freestyle is 51.2 s. Her record for the 200-m freestyle is 1 min 52.8 s. If Suki could swim the 200-m race at the same rate as the 100-m race, how much time would she cut from her record?

18. What number will have a remainder of 1 when divided by 2, 3, 4, or 6, but will have no remainder when divided by 7?

Check your answers by turning to the Appendix, page 188.

Looking Back

In this activity, you used proportional reasoning to solve ratio and rate problems. You then solved percent problems. These are skills you will use frequently in your life to help make consumer decisions.

19. In your journal, complete the following question.

    Larisa has saved for a mountain bike that is on sale this week for $140 off the regular price of $720. She knows that the mountain bike will not be on sale next week but there will be a "scratch and save" promotion, which will be worth either 10%, 20%, or 30% off everything in the store. Determine whether it would be better to buy the bike this week or next week. Provide an argument for your decision.

Check your answers by turning to the Appendix, page 190.

Conclusion

In this section, you distinguished between a ratio and a rate. You wrote rates and ratios in different ways and simplified them whenever possible. You explored percents. Finally, you used proportional reasoning to solve ratio, rate, and percent problems.

To describe business or corporate performance, rates, ratios, and percents are indispensable tools. For example, did you know that VIA Rail, the Crown corporation formed in 1978 to transport passengers between Canadian centres, served over 3.8 million travellers in 1999? Approximately 85% of VIA Rail's ridership was in the Quebec City-Windsor corridor; these passengers accounted for 70% of its income. Also, 300 out of VIA Rail's 460 trains that are scheduled each week travel along this corridor.

Assignment

Turn to Assignment Booklet 2B and complete the assignment for Section 2.

SECTION 3

Data, Bivariate Data, and Scatter Plots

Is there any relationship between the height of an individual and his or her shoe size? Certainly as you grow older and taller, your feet continue to grow until you are mature. But, does a tall person have larger feet than a short person? Can you predict a person's shoe size if you know his or her height or vice versa?

Would a relationship more likely exist between the height of an individual and the height of his or her father? Or would a stronger relationship exist with the height of his or her mother?

These and other relationships can be explored by conducting controlled surveys or by properly designing and carrying out experiments. This is how scientists, medical researchers, and people in many other occupations discover relationships.

In this section, you will design experiments so you can discover the information you are seeking. Then you will organize this data on scatter plots and interpret what the scatter plots show. You will become adept at drawing lines of best fit on your scatter plots and using them to make predictions.

Activity 1: What Is Statistics?

Did you know that the telephone was invented by a Canadian by the name of Alexander Graham Bell?

Here are some statistics about telephones:

- By the end of 1991, there were about 537 000 000 telephone subscribers in the world.
- The busiest international telephone route is between the United States and Canada. In 1991, there were 33 000 000 000 minutes of two-way traffic between these two countries.
- Monaco has 810 telephones for every 1000 people. This country has the most telephones per person.

A statistic is an item of information. A number of pieces of information which have been collected and recorded are called statistics.

The branch of mathematics that deals with the systematic collection and organization of numerical information is also called statistics.

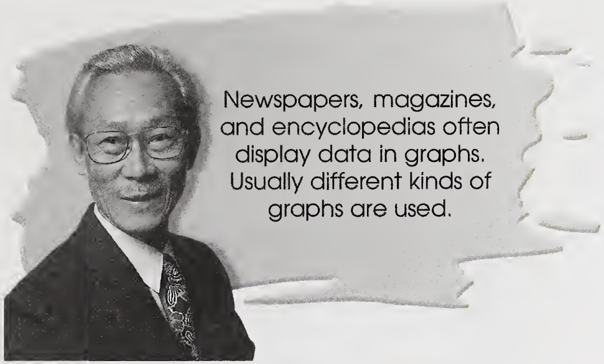
Statistics Canada is the Canadian agency that collects, analyses, and publishes data about Canadians. The publications include pamphlets, books, compact discs (such as *E-STAT*), and articles and databases on the Internet.

A computer database is a collection of data organized for rapid search and retrieval. A database may also be called a data bank.

Statistical information can be found in most libraries. Your library may have the following sources of data:

- encyclopedias and atlases, including multi-media versions
- Statistics Canada publications (such as *Canada Year Book*)
- newspapers and magazines
- books (such as *The Guinness Book of Records*)
- almanacs (such as *The Canadian Global Almanac*, *The World Almanac and Book of Facts*, or *The Information Please Sports Almanac*)
- the Internet

Reading Graphs



Pictographs and bar graphs usually compare the number or size of people or things. The data may be arranged alphabetically, geographically, or in order by size.

Circle graphs and histograms show how a set of data is distributed. The data in circle graphs are often arranged alphabetically. The data in histograms are usually arranged in order of time or in order of size.

Broken-line graphs usually show how a set of data changes over time.

Use the given graphs to answer the following questions.

1.

a. What is being compared in this graph?

b. How is the data in this pictograph arranged?

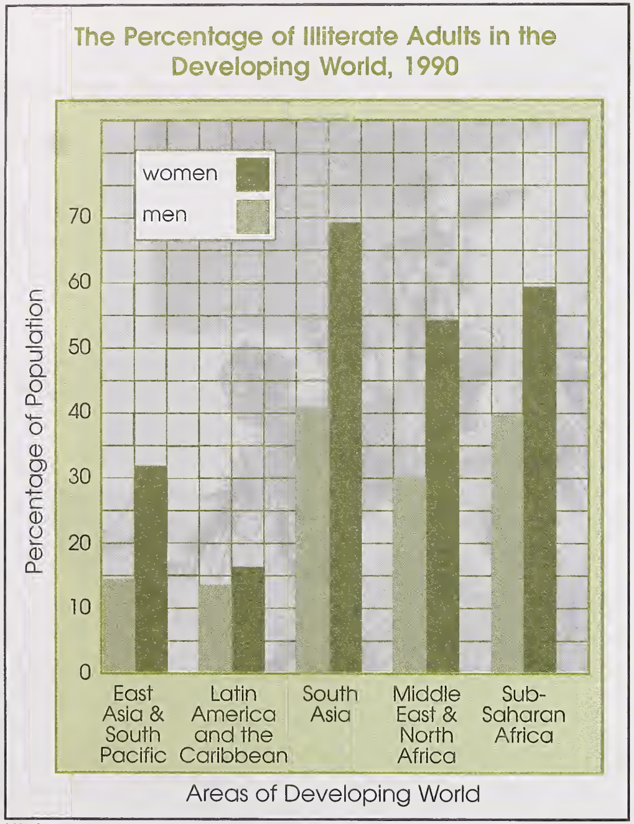
c. If represents 100 000 passengers, what do , , and represent?

d. About how many passengers used Toronto's airport?

e. About how many passengers used Vancouver's airport?

f. About how many more passengers used Toronto's airport than Winnipeg's airport?

2.



SOURCE OF DATA: CIDA

a. What is being compared in this bar graph?

b. In 1990, which area of the developing world had the greatest percentage of illiteracy for men? for women?

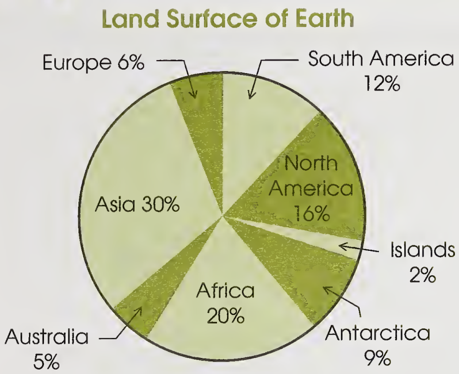
c. In 1990, which area of the developing world had the least percentage of illiteracy for men? for women?

d. According to the graph, how did illiteracy for men compare to illiteracy for women in these areas of the developing world?

e. In 1990, which area of the developing world had the greatest difference in the percentage of men and women who were illiterate?

f. In 1990, which area of the developing world had the least difference in the percentage of men and women who were illiterate?

3.



a. What is being shown in this circle graph?

b. Which continent has the greatest amount of land?

c. Where does North America rank in respect to the amount of land?

d. If there are about 155 400 000 km^2 of land on Earth, find the number of square kilometres of land in North America.

4. The following chart gives the result of a survey.

Newspaper Reading Habits of Junior High Students

Newspaper Section	Percent
Local and Regional News	34
National News	19
International News	17
Editorial/Letters to the Editor	6
Business	3
Sports	38
Arts and Entertainment	42
Comics	56
Homemaking	8
Classified Ads	12

a. Which section of the newspaper is the total population of junior high students most likely to read?

b. Which section of the newspaper is the total population of junior high students least likely to read?

c. Of the 12 000 junior high students in the city, how many are likely to read the sports section?

d. Of the 12 000 junior high students in the city, how many are likely to read the editorial/letters to the editor section?

Check your answers by turning to the Appendix, page 190.

Looking Back

In this activity, you answered questions about data presented in graphs or tables. You also used your skills in working with fractions and percents.

5.

Age of Mother at Birth of First Child*

Age of Mother	1931	1950	1971	1990
Under 15	14	15	292	233
15–19	9 639	14 251	33 258	19 375
20–24	25 224	41 018	65 618	48 555
25–29	13 826	24 330	32 918	68 647
30–34	4 802	8 558	7 236	30 629
35–39	1 580	3 086	1 830	7 338
40–44	342	677	380	828
45 and over	27	37	15	14
Total first-borns**	55 486	92 018	142 008	175 636

* Excludes Newfoundland
** Includes births for which age of mother is not stated

SOURCE OF DATA: STATISTICS CANADA

a. What age interval had the highest number of first-borns in 1931? in 1950? in 1971? in 1990?

b. Calculate the percent of births for mothers under 19 in 1931, in 1950, in 1971, and in 1990. Round to the nearest percent.

c. How many babies were born in 1931 where the birth age of the mother was not stated?

d. Why do you think data for Newfoundland were not involved?

Check your answers by turning to the Appendix, page 192.

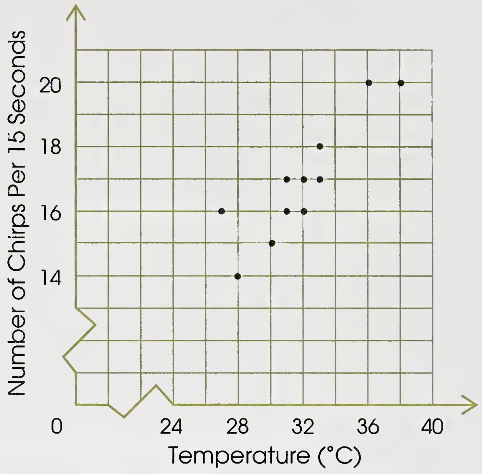
Activity 2: Examining Bivariate Data

Jolene does research for a government agency. In one area of her research, she has data on the number of cricket chirps versus air temperature. One set of data is as follows.

Temperature (°C)	Number of Chirps Per 15 Seconds
28	14
31	16
32	17
30	15
33	17
32	16
36	20
31	17
27	16
38	20
33	18

To graph the data, Jolene puts the number of chirps per second on the vertical axis and the air temperature on the horizontal axis. The resulting graph shows bivariate data, the relationship between the two measures.

Number of Chirps Versus Temperature



Bivariate data is data that involves a relationship between two measures.

Other examples of bivariate data are as follows:

- age group and number of traffic violations
- jogging and length of life
- amount of television watched and school marks

Look at the two measures in the cricket example (temperature and the number of chirps per 15 seconds). Each of these measures is a variable. The graph shows the relationship to be one where if one variable increases, so does the other.

Consider the two variables in terms of cause and effect.

Which variable causes the other to change?

Logic tells you that the number of chirps per 15 seconds cannot be causing a rise in air temperature. It is a logical possibility that warmer air temperature will cause a change in behaviour in a cricket, such as increasing its number of chirps per 15 seconds. In this example, the air temperature is the independent variable.

The variable that causes a change in the other variable is called the independent variable.

The number of chirps per 15 seconds is the dependent variable.

The variable that responds or is changed depending on how the independent variable changes, is called the dependent variable.

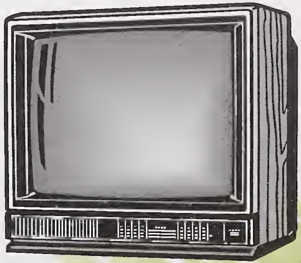
1. State three examples of bivariate data other than those listed previously.

Check your answer by turning to the Appendix, page 192.

You can design and conduct experiments that illustrate the relationships of bivariate data. Experiments differ from surveys in that you can control the conditions or variables rather than just collecting data.

Example

Design an experiment to see if there is a relationship between students' school marks and the amount of television they watch.



Solution

You need to set up a situation where you can reduce the influence of other factors as much as possible. Factors such as age, academic ability, and grade level should be the same or similar. Once the sample has been chosen, keep track of each student's marks and the amount of time spent watching television every day.

When planning any experiment, you should be very clear of its purpose. Choose the two variables that you are going to measure, and set up the plan that your experiment is going to follow.

2. Ms. Coulson teaches a Grade 10 math class at Midtown High School. She wants to show the students how their work habits influence their grades.

a. How might Ms. Coulson design an experiment to see if there is any relationship?

b. What things would Ms. Coulson have to be cautious of or careful with when setting up her experiment?

3. **a.** How could you check to see if there is a relationship between a person's height and his or her shoe size?

   b. What other factors might influence the data?

Check your answers by turning to the Appendix, page 192.

It is not an easy task to design an experiment that will explore the relationship between only two variables. There are always other factors that will have some influence on the results. You have to be aware of these influences when you are designing an experiment and when you are analysing the results.

4. Pose a question that will be the basis of an experiment exploring the relationship between two variables for each of the following topics.

   a. jogging
   b. smoking
   c. fertilizing
   d. studying

5. What two variables would you be measuring in order to answer each of the following questions?

   a. Do people who receive flu shots have fewer incidents of flu?

   b. Do people who weigh more at birth live longer?

6. Identify the independent and dependent variables in questions 5.a. and 5.b.

7. Gather data that could be used to explore each of the following relationships and organize it in a table. Outline how you found the data.

   a. Is there a relationship between the height of a person and his or her hat size?

   b. Is there a relationship between an individual's weight and wrist size?

   c. Is there a relationship between the circumference and the diameter of a circle?

8. The following newspaper article discusses a relationship between two variables.

Better Numbers in Girls' Class

To sum it up, math scores in a new girls-only class at St. Thomas More Catholic junior high school are improving.

The improvement in the girls' marks has ranged from five to 20% since the exclusive Grade 9 class was set up in October, said principal Brenda Willis.

The findings back research that suggest girls will score better in math and sciences without boys in the classroom.

14-year-old Suzanne Kisteleki…went from a 65 percent average in her mixed class at the start of the school year to an 80 percent score on her latest report card.

"It's a lot quieter," said Kisteleki…

Willis convinced the parents' council at St. Thomas More, 9610 – 165 St., to separate two Grade 9 math classes into two single-sex groups. The move was made in late October, after first report cards.

One other Grade 9 class was left mixed with boys and girls. The mixed class and the boys-only group have since shown no noticeable change in marks.

But things began adding up in the girls-only class and interest is compounding.

The girls-only class is taught by a male teacher.

Willis is now planning separate math classes for girls and boys in Grades 7, 8, and 9 next year.[1]

a. Identify the two variables and the relationship that is being discussed.

b. What conclusion, if any, has been drawn?

Check your answers by turning to the Appendix, page 193.

Looking Back

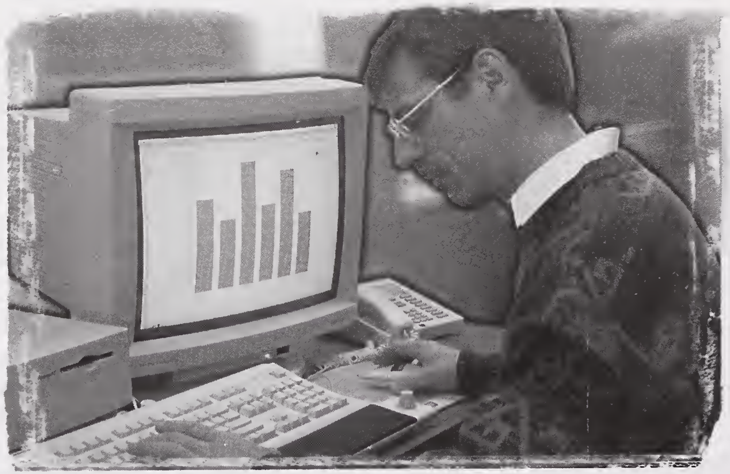
In this activity, you designed and conducted experiments that allowed you to investigate the relationship between two variables.

9. In your journal, design an experiment to investigate the relationship between two variables. Conduct your experiment. Make a chart to record your bivariate data, and analyse the results of your investigation.

Check your answers by turning to the Appendix, page 194.

[1] Timothy le Riche, "Better Numbers in Girls' Class," *The Edmonton Sun*, 5 May 1995. Reprinted by permission.

Activity 3: Making a Scatter Plot

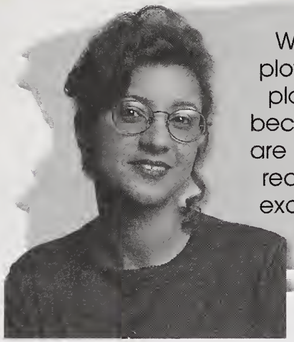


Anybody who conducts an experiment must devise some way of analysing the data in order to discover any relationships or come to any conclusions.

Statistical data can be analysed through manual calculations or using a calculator or a computer. Many of today's statistics are analysed using computers.

In this activity, you will display data involving two variables using a scatter plot.

A scatter plot is a graph of a set of points representing the relationship between two sets of numbers or data. The points are plotted like ordered pairs, and the axes of the graph are labelled appropriately.



When the data is plotted on a scatter plot, patterns may become evident that are not clear or easily recognizable when examining the data alone.

Example

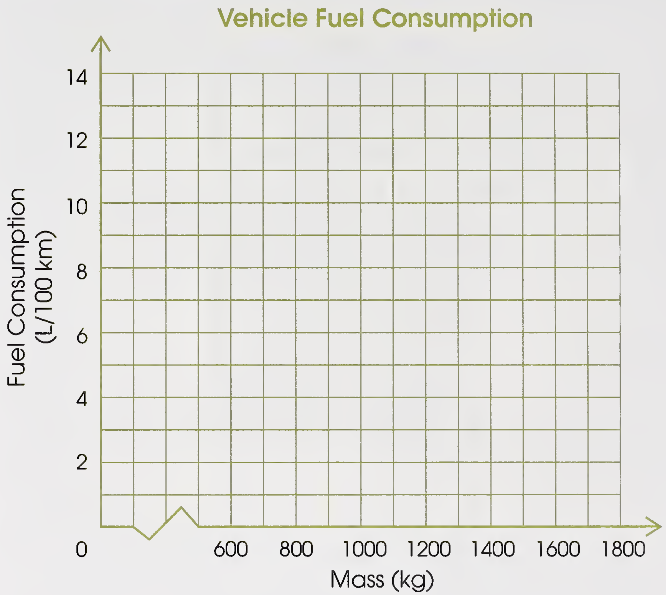
The following data has been collected to determine how the mass of a vehicle affects fuel consumption. Make a scatter plot using the given data.

Vehicle Fuel Consumption

Mass (kg)	Fuel Consumption (L/100 km)	Mass (kg)	Fuel Consumption (L/100 km)
850	5.2	1100	7.5
1200	8.6	1750	12.3
900	6.3	1400	10.5
1050	8.2	1700	12.0
1300	7.9	1000	8.6
600	4.2	1200	8.2
1500	9.8	900	5.7

Solution

Plot the data on a grid similar to the following.

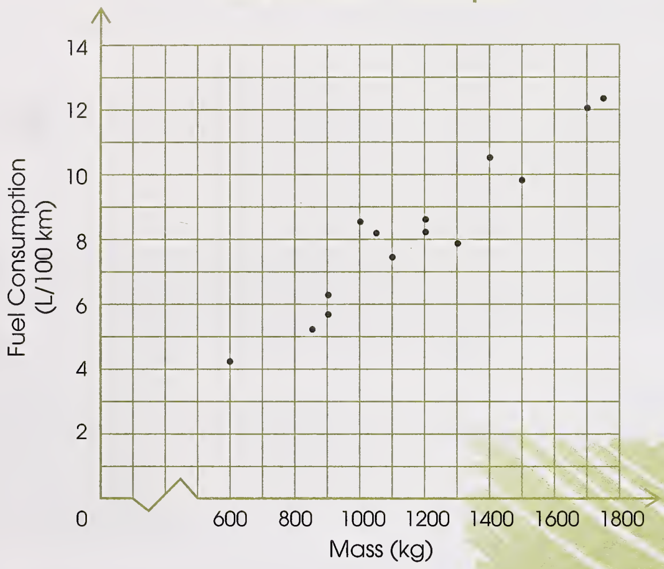


Fuel consumption is dependent on the mass; so, place fuel consumption (the dependent variable) on the vertical axis and mass (the independent variable) on the horizontal axis.

Always place the dependent variable on the vertical axis and the independent variable on the horizontal axis.

The scatter plot will look as follows.

Vehicle Fuel Consumption



Note: Always make sure your scatter plot has a title and that the axes are labelled appropriately.

1. Refer to the scatter plot in the preceding example, describe the relationship between fuel consumption and mass.

Check your answer by turning to the Appendix, page 195.

Drawing a Scatter Plot Using a Computer Spreadsheet

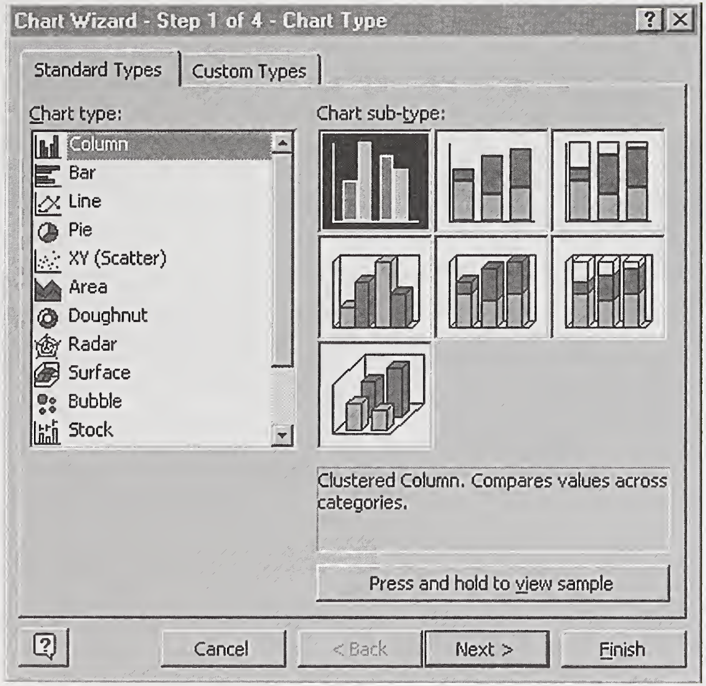
If you have access to a computer spreadsheet program, you can draw a scatter plot using a computer. You can use any spreadsheet program, like *Microsoft® Excel*. Instructions for creating scatter plots in *Microsoft® Excel* follow. You may have to modify the steps somewhat if you are using a different spreadsheet program.

Step 1: Open the *Excel* program, and enter the data given in the previous example for Mass and Fuel Consumption. Enter the data for mass in the first column and the data for fuel consumption in the second column.

	A	B
1	850	5.2
2	1200	8.6
3	900	6.3
4	1050	8.2
5	1300	7.9
6	600	4.2
7	1500	9.8
8	1100	7.5
9	1750	12.3
10	1400	10.5
11	1700	12
12	1000	8.6
13	1200	8.2
14	900	5.7

Step 2: Select cells A1 to B14. Click and hold on cell A1, and drag through to cell B14; then release the mouse button. You should see the highlighted cells.

Step 3: From the Insert menu, choose "Chart… ." The Chart Wizard appears.



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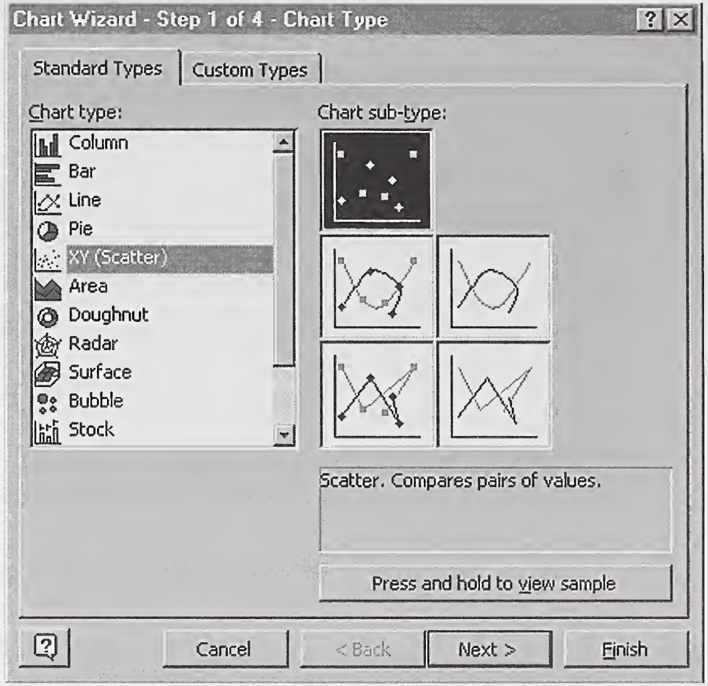
Step 4: Under Chart type, select "XY (Scatter)," and under Chart sub-type, select the scatter plot without lines.



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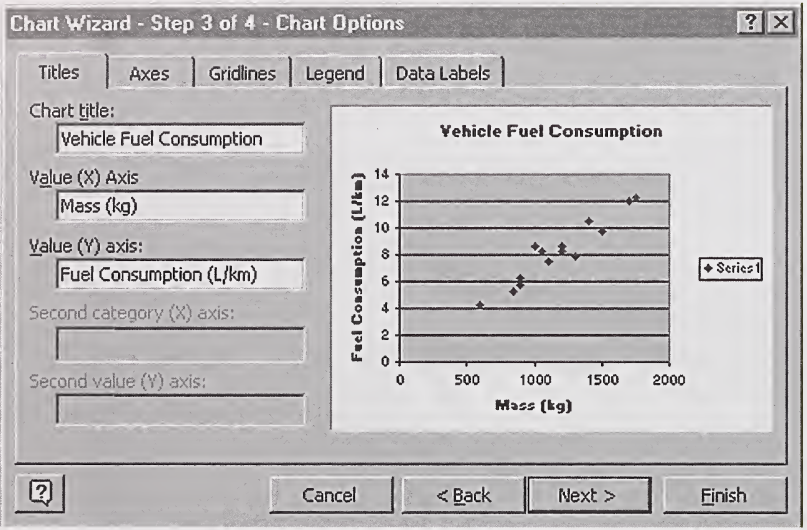
Step 5: Click on "Next" to preview the scatter plot.

Step 6: Click on "Next," and insert the title and axes labels.

- Title the graph "Vehicle Fuel Consumption."
- Label the x-axis "Mass (kg)."
- Label the y-axis "Fuel Consumption (L/100 km)."



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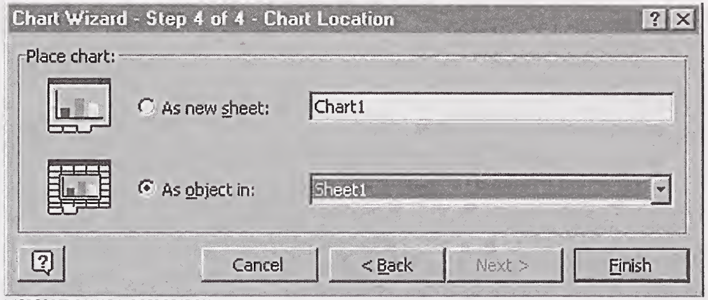
Note: The other tabs allow you to make other modifications to the graph.

Step 7: Click on "Next." Now, choose whether you want the chart to be placed in a document or kept as a separate item. The auto option will place the scatter plot on your spreadsheet. Choose this option. (As a separate spreadsheet, the other option will make a separate page with the scatter plot.)



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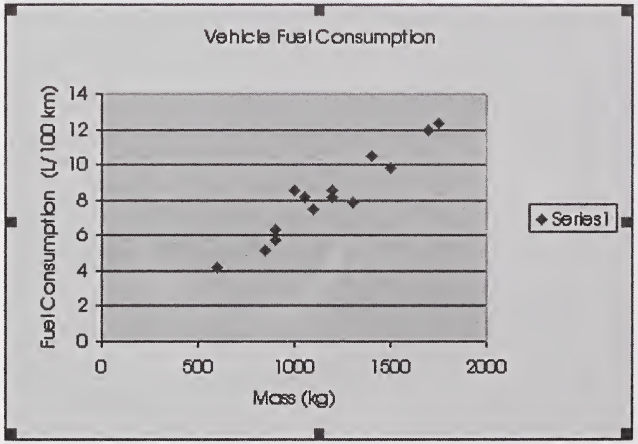
Click on "Finish." You can now print the graph or save it where desired.



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Note: Be sure to save your scatter plot for later use.

Use either paper and pencil or a computer spreadsheet program to answer the following questions.

2. The coach of the Manning Marauders in the Northern Alberta Senior Hockey League recorded the total number of points scored by each player and the average number of minutes played per game. The results of the first half of the season are listed.

Player Number	Number of Points Scored	Amount of Playing Time Per Game (min)
2	12	27
3	18	15
5	32	24
8	20	18
9	9	10
11	30	20
14	13	12
17	48	28
23	28	20
35	36	26
42	21	16
66	27	19
88	35	22
99	54	32

a. Construct a scatter plot of the data.

b. Describe any relationship or trend shown in the graph.

c. Are there any points that don't seem to fit? If so, give an explanation as to why they don't fit.

3. Make a scatter plot of the following data, and note any trend. Comment on the trend you notice.

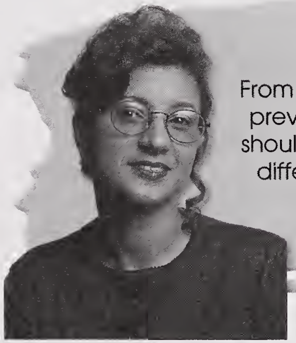
Average Monthly Temperature of Two Cities	
Melbourne, Australia (°C)	Washington D.C., U.S.A. (°C)
26	7
26	7
24	13
20	18
17	24
14	28
13	30
15	29
17	26
19	19
22	13
24	7

4. Make a scatter plot of the following data, and note any trend.

Daily High Temperature and Relative Humidity

Temperature (°C)	Relative Humidity (%)	Temperature (°C)	Relative Humidity (%)
20	65	25	50
30	16	10	40
16	85	29	62
18	24	15	32
24	10	9	15
8	78	22	58

Check your answers by turning to the Appendix, page 195.



From your answers to the previous questions, you should have found three different relationships.

Trends or Relationships in Scatter Plots

- Points plotted indicate that when one measure increases, the other measure also increases.
- Points plotted indicate that when one measure increases, the other measure decreases.
- Points show no apparent relationship.

5. What type of relationship would you expect for each of the following?

   a. age of spouses

   b. number of years of post-secondary education and income

   c. hat size and sense of humour

   d. temperature outside and amount of ice cream sold

   e. speed travelled and time taken to reach a destination

Check your answers by turning to the Appendix, page 196.

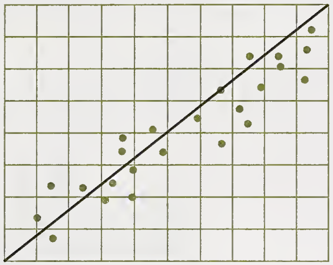
Looking Back

In this activity, you created scatter plots and looked for trends or relationships between the variables.

6. Look through the newspaper or magazines and try to find data that has been represented by scatter plots.

Activity 4: Drawing Lines of Best Fit and Analysing Scatter Plots

When a researcher creates a graph from research data, he or she will often connect the points to form a line or draw a line through as many points as possible.



In the last activity, you looked at scatter plots to see if there were any general trends or relationships. In this activity, you will look at how you can make further interpretations from a scatter plot by adding a straight line to the diagram.

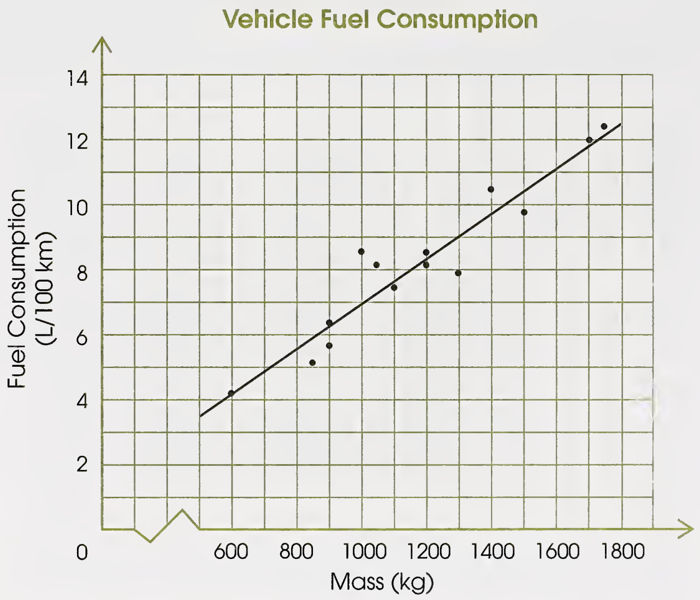
Line of Best Fit

The points on a scatter plot do not fall exactly along a line, but often a line can be drawn that closely approximates the data. This line is called the line of best fit.

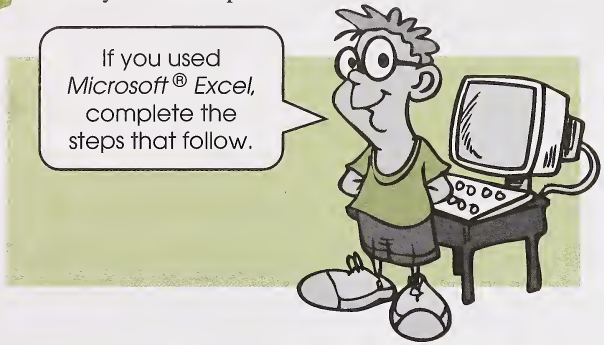
A line of best fit can be drawn by placing your ruler on the graph and drawing a line that appears to best approximate the points on the scatter plot.

Copy the scatter plot you completed for vehicle fuel consumption in Activity 3 of this section.

Use a ruler to draw a line through the points so that approximately half are on one side of the line and half are on the other side of the line. You now have a line of best fit for the data. Compare your line of best fit for vehicle fuel consumption to the one shown on the following scatter plot.

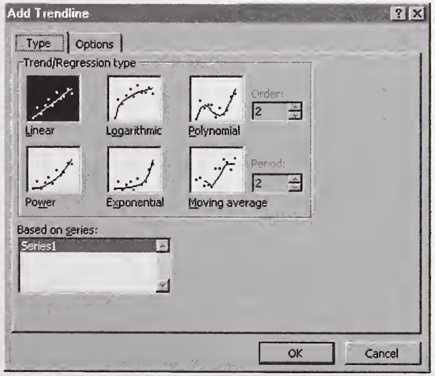


If you used a spreadsheet program to create the scatter plot in Activity 3, you can use that same program to add a line of best fit to your scatter plot.



Open your saved file that has the scatter plot you created. You may want to enlarge your graph. Grab an anchor point, like the one at the lower right-hand corner, and move it down and to the right.

Step 1: From the Chart menu, click on "Add Trendline." The following window appears.



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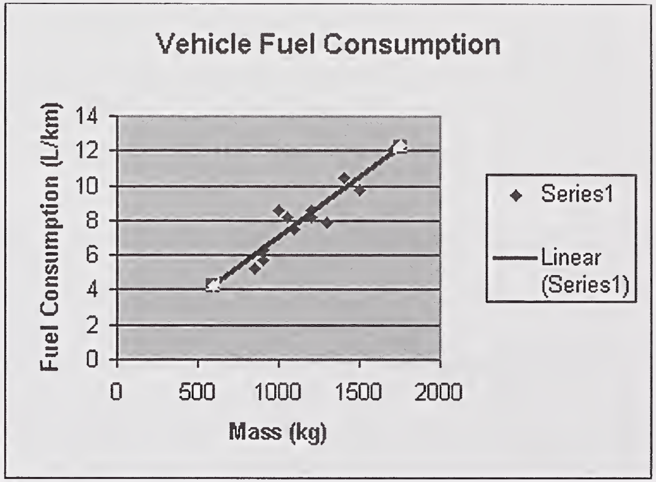
Step 2: Choose "Linear," and click "OK." The trendline (line of best fit) should appear somewhat as follows.



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1. Is the line a fairly good approximation of the points that were plotted?

2. What kind of relationship does the line suggest?

Check your answers by turning to the Appendix, page 196.

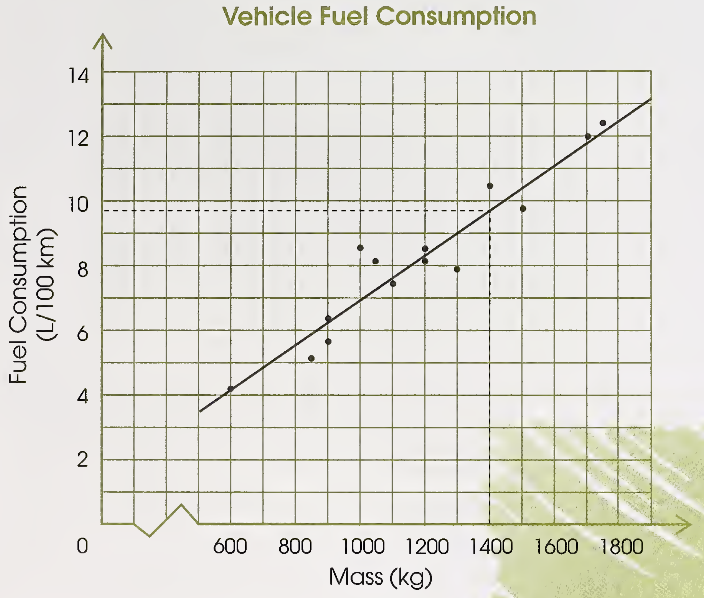
You can use the line of best fit to find values between the given values (interpolation) or before the first value and after the last value (extrapolation). This is useful in predicting new data based on known data.

Example

Use the preceding graph to find the fuel consumption for a minivan that has a mass of 1400 kg.

Solution

Use a ruler to find the point on the line of best fit that is directly above 1400. Go horizontally from that point to the vertical axis. Read the fuel consumption from the vertical axis.



The fuel consumption for a 1400-kg vehicle is about 9.8 L/100 km.

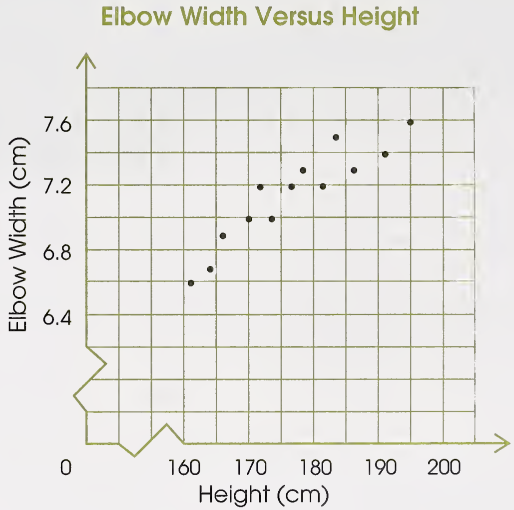
3. Use the graph of vehicle fuel consumption to answer the following questions.

   a. Find the fuel consumption for a vehicle with a mass of 1900 kg.

   b. According to the scatter plot, what would be the mass of a vehicle if it has a fuel consumption of 3.5 L/100 km?

   c. What can you say about the fuel efficiency of the vehicles represented by the points that are not on the line of best fit?

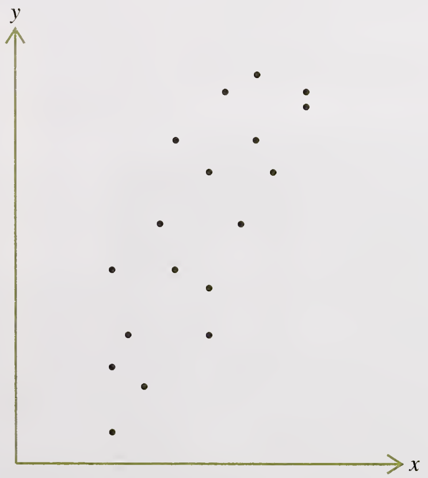
4. Copy the following scatter plot into your notebook; then draw a line of best fit.



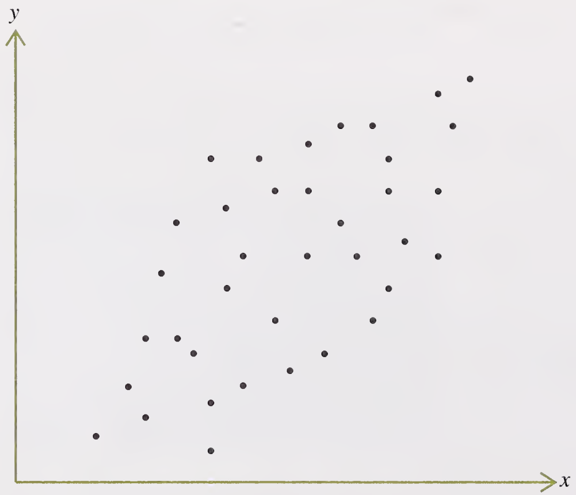
5. Copy the following scatter plots into your notebook; then draw a line of best fit for each.

a.

b.



c.



6. On which scatter plot in question 5 was the line of best fit easiest to locate? Why?

7. The data in the following table shows the age and number of push-ups that could be done by 20 female staff members at Eastside High School.

Age	Number of Push-ups	Age	Number of Push-ups
24	18	60	1
28	14	45	6
54	4	36	12
48	8	53	2
37	11	40	7
30	15	29	12
34	12	39	21
42	6	43	8
46	5	57	3
51	4	47	5

a. Draw a scatter plot of the data; then draw a line of best fit.

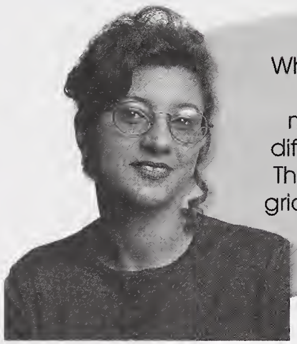
b. What conclusions can you draw from the scatter plot?

c. What might account for the one exception?

d. Did you allow the one point that was an exception to influence your line of best fit?

e. According to the graph, how many push-ups can female staff who are age 60 and over do? Is this necessarily true for society as a whole?

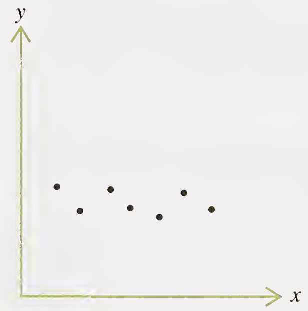
Check your answers by turning to the Appendix, page 196.



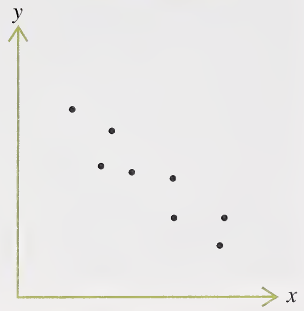
When performing experiments, scientists frequently take measurements to learn how different quantities are related. They plot these quantities on a grid, and the resulting graph is a scatter plot.

8. The following scatter plots are the result of data that was collected.

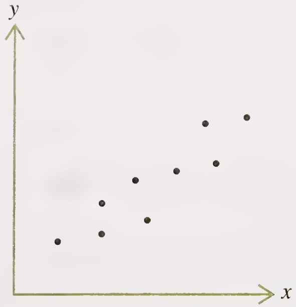
Scatter Plot A



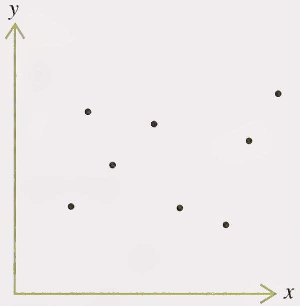
Scatter Plot B



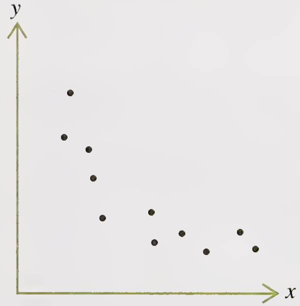
Scatter Plot C



Scatter Plot D



Scatter Plot E



a. Which of the preceding scatter plots show some kind of trend?

b. What kind of line would you draw through the points in scatter plot E?

c. In your notebook, copy scatter plot D and connect the points. What would you conclude about this data? What would you conclude about the relationship between the variables?

9. Design an experiment to measure the height a ball, such as a basketball, bounces versus the height from which it is dropped. Obtain data for at least ten different heights.

a. Make a chart and plot the points on a scatter plot. Then draw a line of best fit.

b. What trend or relationship do you observe from the line of best fit?

c. Why might some points be noticeably further from the line of best fit than others?

d. Use your line of best fit to predict how high a ball dropped from 100 cm would bounce.

e. Will the type of ball (such as a tennis ball, a rubber ball, or a superball) make a difference?

f. If possible, test your prediction in question 9.e. using different balls.

10. Karen, a basketball coach, kept a number of statistics about all of her players for each game in a ten-game period. After the tenth game, Karen analysed the data carefully to see what she might do to improve her team's performance.

a. Two variables that Karen tabulated were the amount of time played per game and the number of points scored per game. Use the data that follows to make a scatter plot; then draw the line of best fit. Label each axis appropriately, and give the scatter plot a title.

Player	Time Played (min)	Number of Points Scored
Muldoon	4	3
Zapisocki	7	4
Koupax	10	6
Reusel	12	10
Deen	12	6
Bouvier	16	11
Smithson	17	8
Armdal	17	14
Leader	20	11
House	20	20
Wong	22	14
Rav	24	18
Naidoo	27	18
Ryning	29	12
Wetland	29	22

b. What is the general trend for the scatter plot?

c. What questions might Karen, as a coach, be asking herself about players Ryning and House based on the data in the scatter plot?

d. If a player has an average playing time per game of 25 min, how many points would you expect her to score?

e. If one of the players' playing time was increased to 30 min per game, how many points might you expect her to score?

Check your answers by turning to the Appendix, page 198.

Looking Back

You are now able to draw a line of best fit on a scatter plot and to use that line to make predictions and draw conclusions.

11. In your journal, describe in words the relationship between the line of best fit and the data points that is necessary for making good predictions.

Check your answers by turning to the Appendix, page 200.

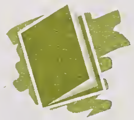
Conclusion

In this section, you have gone through the first four steps of statistics. You collected data from an experiment you designed and organized, then presented that data using scatter plots. You also looked at ways to analyse data by drawing a line of best fit. Finally, you used scatter plots and lines of best fit to draw inferences or conclusions about the variables involved.

By designing an experiment and obtaining a follow-up analysis, you can discover the relationship, if any, between any two variables. You can determine such relationships as the height of a person to his or her shoe size or to the heights of that person's parents.

Do you think you will be approximately the same height as your parents?

Assignment



Turn to Assignment Booklet 2C and complete the assignment for Section 3.

Module Summary

In Section 1, you developed a number sense for fractions. You added, subtracted, multiplied, and divided fractions and mixed numbers; then you practised using a calculator for performing these operations on fractions.

In Section 2, you applied your knowledge of fractions to ratios and the variety of ways they can be expressed. You also explored rates and percents, and you applied your skills in this section to practical problems.

Lastly, in Section 3, you worked with statistics, looking for patterns in data. You then portrayed bivariate data on scatter plots, drew lines of best fit, and explained the types of relationships shown by the graphs.

The mathematical concepts you worked with in Module 2 are some of the most commonly used in day-to-day living. Just think how many situations involve the use of fractions, ratios, rates, and percents. Sales and discounts, for example, almost always involve percents, as does working out your school marks. Also, think of all the situations in a series of car races that could be expressed as fractions, ratios, proportions, percents, and bivariate data.

Appendix

Problem-Solving Strategies
Glossary
Suggested Answers
Credits

Problem-Solving Strategies

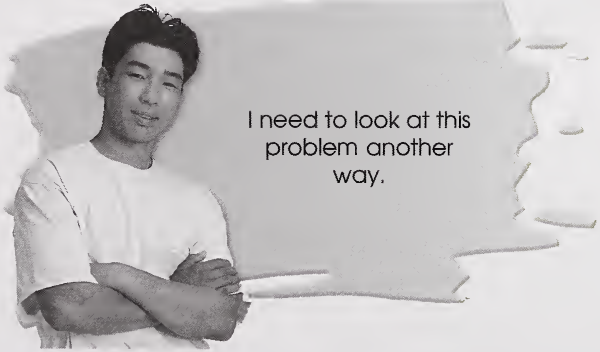
Be sure to study the following problem-solving strategies and use them when you encounter problems in this course.

Whichever method of problem solving you may choose to use, consider the following:

- Show that you understand the problem by showing all the steps needed for finding the answer.
- Draw and neatly label any diagram, graph, or chart that may help you answer the problem.
- Write a concluding sentence that answers the question being asked.

Changing Your Point of View

Sometimes you find that you cannot solve a particular problem because of your "mind set." Perhaps you made an assumption about the problem that is incorrect. If your attempts to solve a problem are not successful, it is often helpful to try to change your point of view.

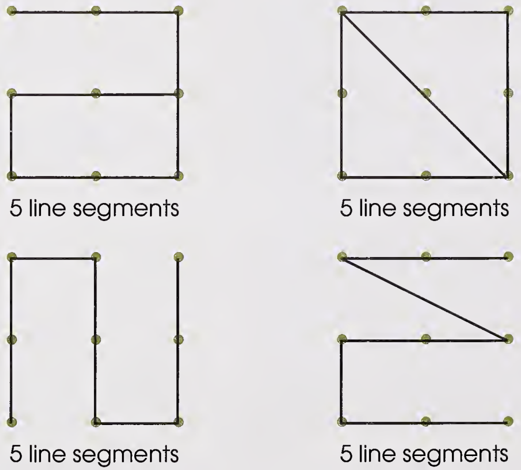


Example

Draw four line segments through the following nine dots without lifting your pencil from the paper and without retracing your path.

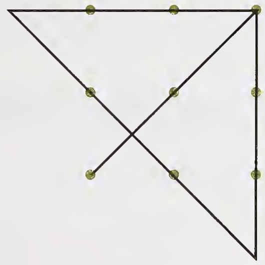
Solution

Step 1: You make several attempts at solving the problem.



Step 2: You realize you are blocked in your attempts. You have made the assumption that the dots must be connected in a way similar to a child's dot-to-dot picture.

In order to solve the problem, you must change your point of view and realize that the line segments may lie outside the confines of the dots.



4 line segments

Using Objects

Objects can be used to help you solve a problem.

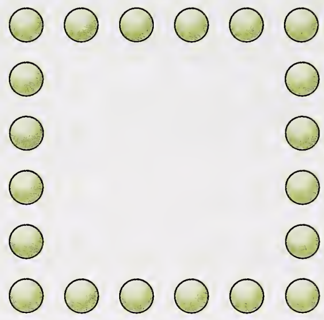
Example

To enclose a square lot with a fence, 20 posts are used. If the posts are placed 3 m apart, what is the length of one side of the lot?

Solution

Step 1: Use objects such as blocks, checkers, or coins to represent the posts.

The advantage of using objects to represent the posts is that you can easily rearrange them.



Step 2: Count the number of spaces between the posts on one side of the square.

There are five spaces between the six posts on one side of the square.

Step 3: Calculate the length of one side of the square.

$$5 \times 3 = 15$$

The length of one side of the lot is 15 m.

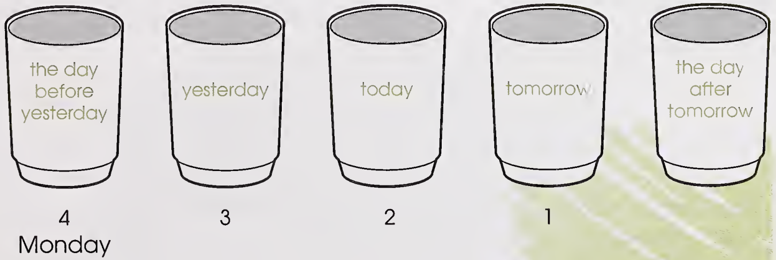
Example

Four days before the day after tomorrow was Monday. What day of the week is it today?

Solution

Step 1: Use paper cups to represent the days. The cups could be labelled as shown.

Step 2: Begin at the cup representing the day after tomorrow and count back four days. That day was Monday.



If Monday was the day before yesterday, yesterday was Tuesday, and today is Wednesday.

Using Diagrams

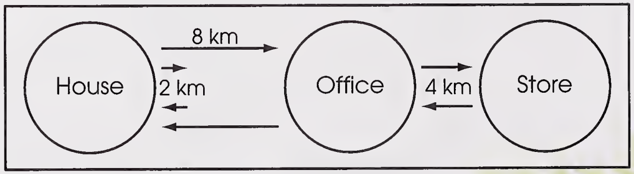
You can use sketches to solve problems.

Example

Chris's office is 8 km from his home. Yesterday morning he drove 2 km before he realized that he had forgotten his briefcase. He returned home to get his briefcase and then drove to his office. At noon, Chris drove 4 km to a store. He then went back to the office for the rest of the afternoon. At the end of the day, he drove straight home. How far did he drive yesterday?

Solution

Step 1: Draw and label a diagram to help you understand the problem.



Step 2: Calculate the distance Chris drove.

$$2 + 2 + 8 + 4 + 4 + 8 = 28$$

Chris drove 28 km.

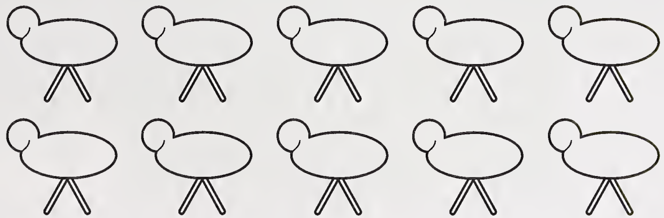
Example

There are 10 animals in a barnyard. Some are chickens and the rest are sheep. Ruth counts 28 legs. How many chickens and how many sheep are there? **Hint:** Chickens have 2 legs and sheep have 4 legs.

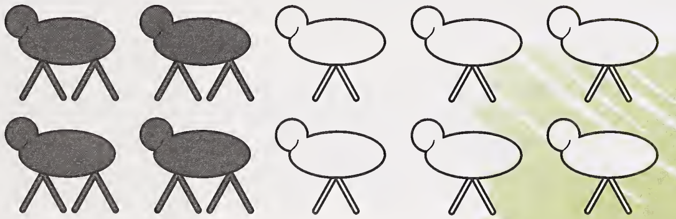
Solution

Draw a diagram to help you solve the problem.

Step 1: Every animal has at least 2 legs. So, draw a diagram of 10 animals, each of which has at least 2 legs.



Step 2: The group of animals in the problem had 28 legs. So, you need to add 8 legs to the drawing. Add the legs in pairs to the sketch of each animal.

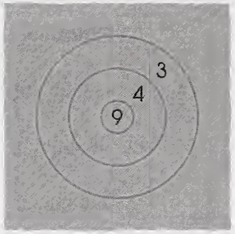


There are 4 sheep and 6 chickens.

Making an Organized List

Some problems require you to list all the possible solutions. It is important to make an organized list so that you do not miss any possibilities.

Example



Julio and his friends like playing a game called Bull's Eye with lawn darts. They draw a target in the dirt and assign values to the three regions of the target.

If Julio throws three darts and all three hit the target, how many different point totals are possible?

Solution

Begin with the highest possible score and list all the possibilities in order.

$9+9+9=27$ ← highest possible score

$9+9+4=22$

$9+9+3=21$

$9+4+4=17$

$9+4+3=16$

$9+3+3=15$

$4+4+4=12$

$4+4+3=11$

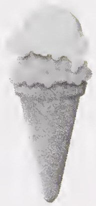
$4+3+3=10$

$3+3+3=9$ ← lowest possible score

There are 10 possible point totals.

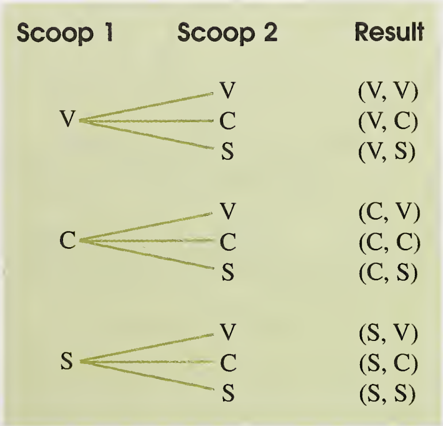
Example

At an ice-cream parlour there are 3 flavours of ice cream: chocolate, vanilla, and strawberry. How many different double-scoop ice-cream cones are possible? **Hint:** A chocolate-vanilla cone is different from a vanilla-chocolate one. Order makes a difference.



Solution

Draw a tree diagram to help you make an organized list. Use V for vanilla, C for chocolate, and S for strawberry.



Nine different double-scoop ice-cream cones are possible.

Using Venn Diagrams

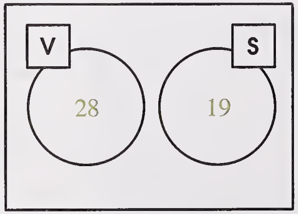
A type of diagram that is helpful in some problems is a Venn diagram. In this special kind of diagram, circles are used to represent groups of people, animals, or objects that have certain characteristics. The positioning of the circles in relation to one another represents relationships among these groups. These diagrams were named after an English mathematician by the name of John Venn (1834–1923).

Example

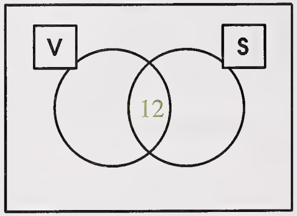
There are 28 girls on the volleyball team and 19 girls on the swim team. If 12 girls belong to both teams, how many girls belong only to the swim team?

Solution

Step 1: Represent the members of the volleyball team with a circle. Represent the members of the swim team with another circle.



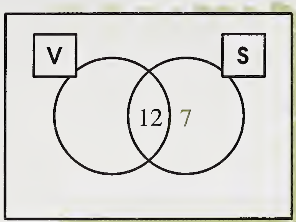
Step 2: Represent the members of both teams by the intersection of the two circles.



Step 3: Now, calculate the number of girls who belong only to the swim team.

$$19 - 12 = 7$$

So, 7 girls belong only to the swim team.

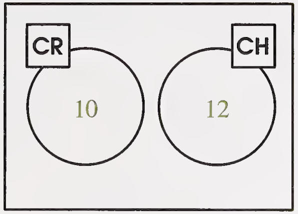


Example

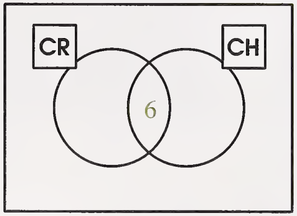
Reilly bought some plates at a garage sale. He discovered that 10 plates were cracked, 12 plates were chipped, 6 plates were both chipped and cracked, and 2 plates were neither chipped nor cracked. How many plates did Reilly buy?

Solution

Step 1: The problem states that 10 plates were cracked (CR) and 12 plates were chipped (CH). Represent the cracked plates with a circle. Represent the chipped plates with a second circle.

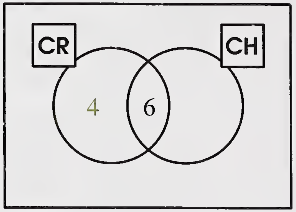


Step 2: The problem states that 6 plates were both chipped and cracked. Represent the cracked and chipped plates with the intersection of the two circles.



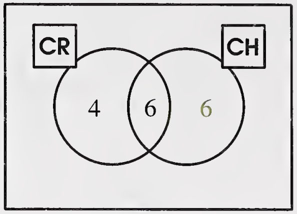
Step 3: Calculate the number of plates that were only cracked.

$$10 - 6 = 4$$

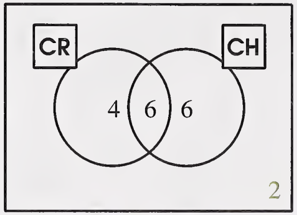


Step 4: Calculate the number of plates that were only chipped.

$$12 - 6 = 6$$



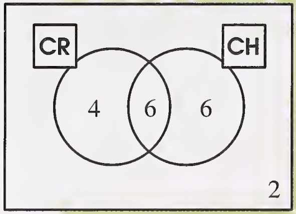
Step 5: The problem states that 2 plates were neither chipped nor cracked. Represent the undamaged plates with the region outside the circles.



Step 6: Calculate the total number of plates that were bought.

$$4 + 6 + 6 + 2 = 18$$

So, 18 plates were bought.



Example

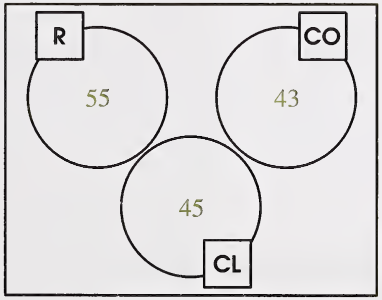
In a survey of 100 people, the following information was collected:

- 43 people like country music
- 55 like rock music
- 45 like classical music
- 8 people like all three forms
- 15 people like both country and classical music
- 25 like both rock and country music
- 20 like both rock and classical music

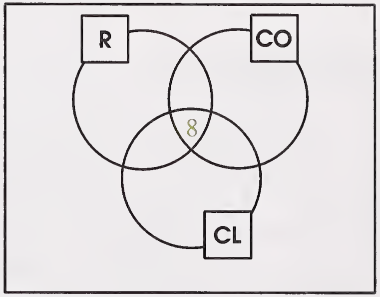
How many people do not like any of these forms of music?

Solution

Step 1: The problem states that 55 people like rock music (R), 43 like country music (CO), and 45 like classical music (CL). Use three circles to represent these preferences.

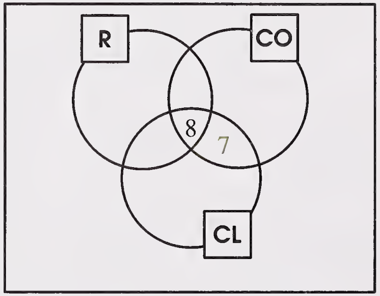


Step 2: The problem states that 8 people like all three forms. The intersection of the three circles shows the people who like all three forms.



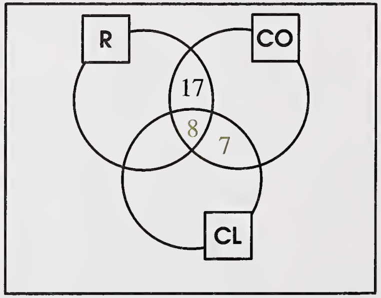
Step 3: The problem states that 15 people like both country and classical music. Calculate the number of people who like country and classical music, but not rock music.

$$15 - 8 = 7$$



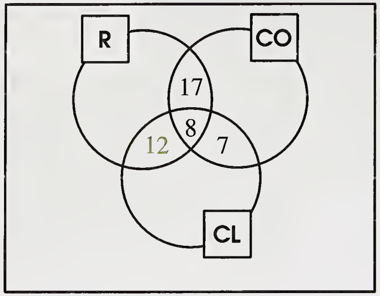
Step 4: The problem states that 25 people like both rock and country music. Calculate the number of people who like both country and rock music, but not classical music.

$$25 - 8 = 17$$



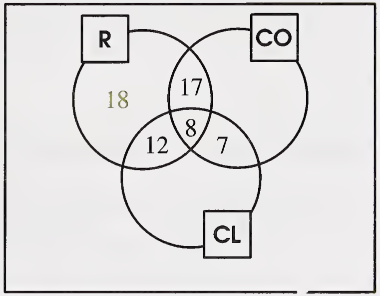
Step 5: The problem states that 20 people like both rock and classical music. Calculate the number of people who like both rock and classical music, but not country music.

$$20 - 8 = 12$$



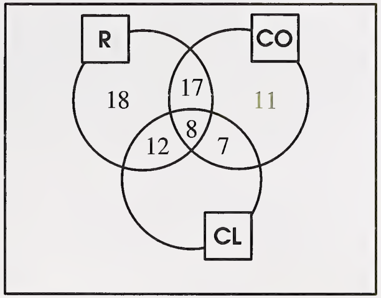
Step 6: Now that you know the number of people who like rock music and any other form of music, calculate the number of people who only like rock.

$$55 - (17 + 8 + 12) = 18$$



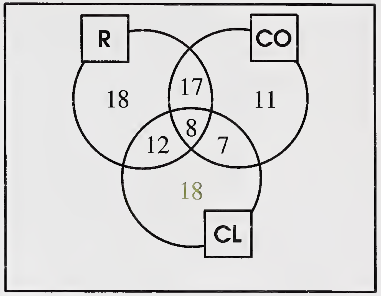
Step 7: Now that you know the number of people who like country music and any other form of music, calculate the number of people who only like country music.

$$43 - (17 + 8 + 7) = 11$$



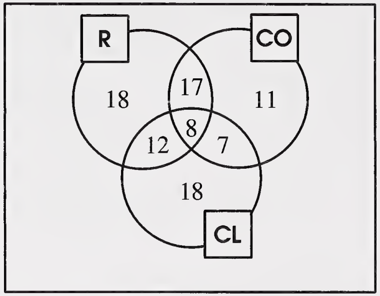
Step 8: Now that you know the number of people who like classical music and any other form of music, calculate the number of people who only like classical music.

$$45 - (12 + 8 + 7) = 18$$



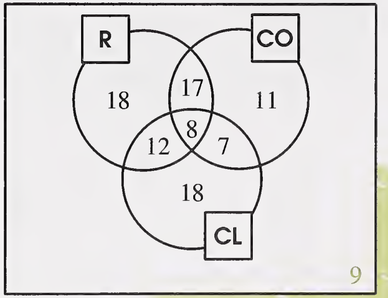
Step 9: Now, you can calculate the total number of people who like rock, country, or classical music.

$$18 + 17 + 8 + 12$$
$$+ 11 + 7 + 18 = 91$$



Step 10: The problem states that 100 people were surveyed. So, you can calculate the number of people who do not like any of these forms of music.

$$100 - 91 = 9$$



Of the 100 people surveyed, 9 people do not like any of these forms of music.

Making a Table

You can use a table to organize information.

Example

Hannah works at her mother's store after school. She earns 20¢ for each customer that she assists and 35¢ for each bag that she packs. One day she did 19 jobs and earned \$4.85. How many bags did she pack and how many customers did she assist that day?

Solution

Write down all possible combinations of 19 jobs until you find a combination that totals \$4.85. Use a table to organize the information.

In the first column, put 1 bag packed and 18 customers assisted. Calculate the money received for that.

$$\begin{array}{r} 1 \times 35¢ = \$\ \ .35 \\ \underline{18 \times 20¢ = \$3.60} \\ \text{Total} \quad \$3.95 \end{array}$$

Then complete the table for other combinations.

Packing	1	2	3	4	5	6	7
Assisting	18	17	16	15	14	13	12
Earned	\$3.95	\$4.10	\$4.25	\$4.40	\$4.55	\$4.70	\$4.85

correct guess

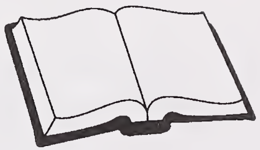
Hannah packed 7 bags and assisted 12 customers that day.

Guessing, Checking, and Revising

Many mathematical problems can be solved by systematic trial or guessing, checking, and revising.

Example

Shelley opened her math textbook and noticed that the product of the numbers of the two open pages was 2970. To what pages was the book opened? **Hint:** The left-hand page number of a book is even; the right-hand page number is odd.



Solution

Method 1: Using Paper and Pencil

Step 1: Note the conditions that need to be met.

- The left-hand page number must be even.
- The right-hand page number is one more than the left-hand page number.
- The product of the numbers must be 2970.

Step 2: Make a guess; check your guess; and revise if necessary.

It is helpful to organize your guesses in a table.

Left-Hand Page Number	50	52	54
Right-Hand Page Number	51	53	55
Product of Page Numbers	2550	2756	2970

correct guess

So, the book was opened to pages 54 and 55.

Method 2: Using a Computer

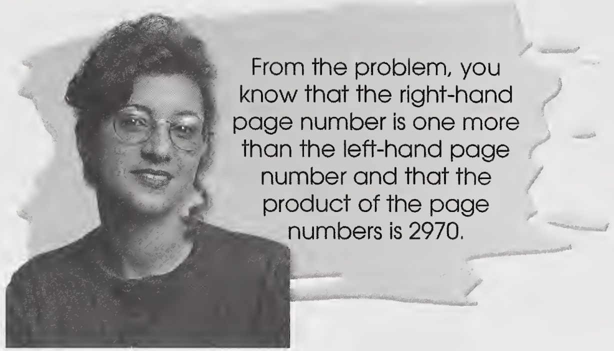
Use a computer and a spreadsheet program to help you solve the problem.

Step 1: Label the cells in the spreadsheet. **Hint:** The cell address at the top of the spreadsheet shows which cell is active. There is also a border around the active cell. You don't type directly in this cell; instead, you use the entry bar at the top of the spreadsheet.

A5 | ✗ | ✓ | Product of page numbers

	A	B
1	**Book Problem**	
2		
3	Left-hand page number	
4	Right-hand page number	
5	Product of page numbers	
6		

Step 2: Enter either a number or a formula for each of the cells.

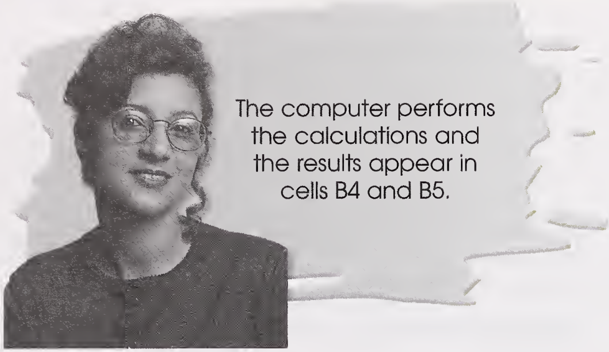


Here is what you enter:

- Begin with cell B3 and enter a guess of 50.
- Next, enter "=B3+1" in cell B4. This means the number in the B4 cell will be one more than the number in the B3 cell.
- Finally, enter "=B3*B4" in cell B5. This means that the number in the B5 cell will be the product of the numbers in the B3 and B4 cells.

B5 | ✗ | ✓ | =B3*B4

	A	B
1	**Book Problem**	
2		
3	Left-hand page number	50
4	Right-hand page number	51
5	Product of page numbers	2550
6		



Step 3: Check your initial guess, and revise it if necessary. The product is 2970, not 2550; therefore, revise your guess.

Delete the number in the B3 cell and enter a guess of 54.

B3 | ✗ | ✓ | 54

	A	B
1	**Book Problem**	
2		
3	Left-hand page number	54
4	Right-hand page number	55
5	Product of page numbers	2970
6		

The computer performs the calculations and different results appear in cells B4 and B5.

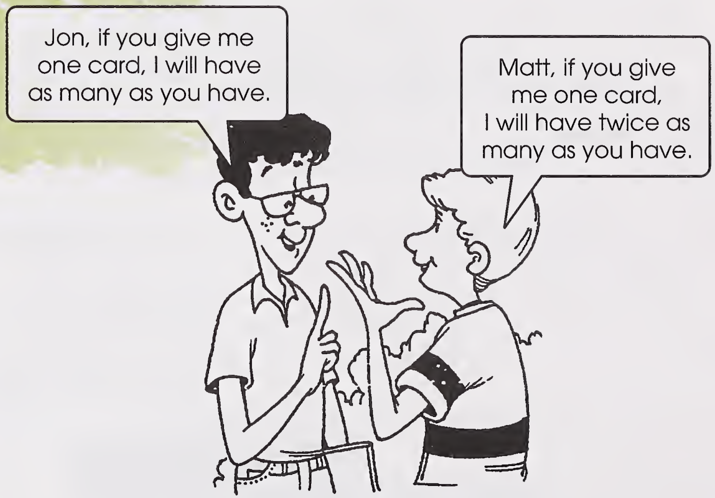
Step 4: Check your initial guess, and revise it if necessary. The product is 2970.

So, the book was opened to pages 54 and 55.

Acting Out a Problem

For some problems, you may find it helpful to physically act out the problem situation.

Example



How many cards does each boy have?

Solution

Step 1: Ask yourself these questions:

- What would happen if Jon gave Matt a card? The first condition to be met is that they must have an equal number of cards after the exchange.

- What would happen if Matt gave Jon a card? The second condition to be met is that Jon will have twice as many cards as Matt after the exchange.

Step 2: Make a guess and act out the problem. If your guess does not fit the conditions, revise your guess and repeat the process.

It is helpful to organize your guesses in a table.

Matt's cards	1	2	3	4	5
Jon's cards	3	4	5	6	7

correct guess

So, Matt has 5 cards and Jon has 7 cards.

Note

If Jon gives Matt a card, each boy will have an equal number of cards after the exchange, 6.

If Matt gives Jon a card, Jon will have twice as many cards as Matt after the exchange. (Jon will have 8 cards and Matt will have 4 cards.)

Working Backwards

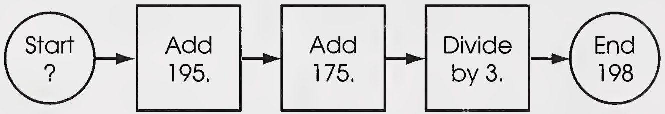
In mathematics, you are usually given information and then asked to find the answer. Sometimes, however, you are given the answer and then asked to find a piece of information. In cases like these, you need to work backwards.

Example

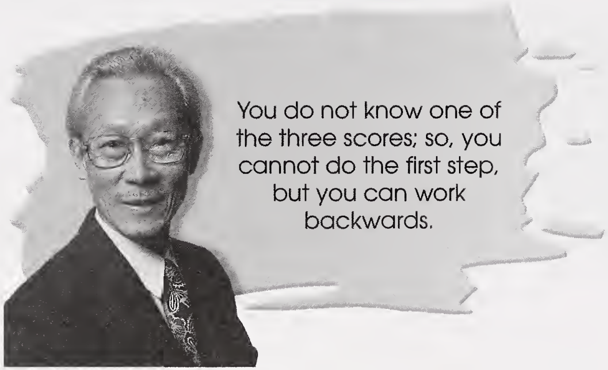
Ross plays in a bowling league. His average in the league is 198. Today Ross scored 195 and 175 in his first two games. What must he score in his third game to maintain his average?

Solution

Step 1: Consider how the problem would be done if you worked forwards. This flow chart shows the sequence of the steps.



You start with the score of the third game, add each of the scores of the other two games, divide by 3, and end with the average of 198.



Step 2: Use a reverse flow chart to work backwards.

These operations undo operations in the original flow chart.

End ? ← Subtract 195. ← Subtract 175. ← Multiply by 3. ← Start 198

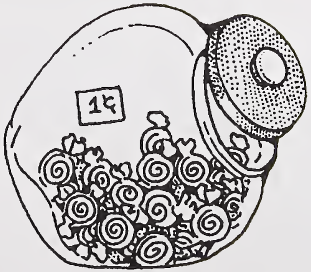
If you start with the average of 198, multiply by 3, subtract 175, and subtract 195, you will end with the score Ross must get in the third game.

Ross must score 224 in the last game to maintain his average.

Note: You can check to see if 224 is the correct third score by using the flow chart in Step 1.

Example

Ruth has a jar of candies. She gives Raschid one-third of her candies. She then gives Steven one-fourth of the remaining candies. Finally, she gives one-fifth of the candies that she has left to Rachel. If Ruth has 12 candies left at the end, how many did she have at the beginning?



Solution

Step 1: Use a flow chart to show the order of operations.

Start ? → Multiply by $\frac{1}{3}$. → Subtract ? → Multiply by $\frac{1}{4}$. → Subtract ? → Multiply by $\frac{1}{5}$. → Subtract ? → End 12

This flow chart is **not** helpful because you do not know how much to subtract in the second, fourth, and sixth boxes.

Knowing the following relationships will help you draw a more complete flow chart.

- Giving away one-third of the candies is the same as multiplying the number of candies by two-thirds.

  $1-\frac{1}{3}=\frac{2}{3}$

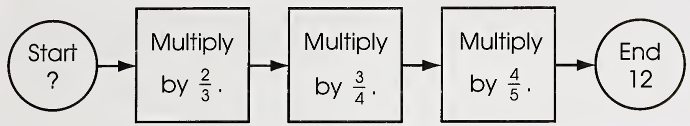
- Giving away one-fourth of the remaining candies is the same as multiplying by three-fourths.

  $1-\frac{1}{4}=\frac{3}{4}$

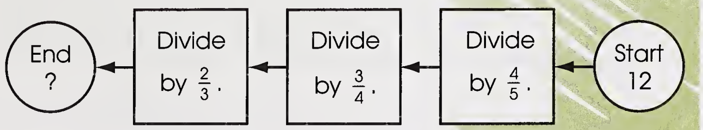
- Giving away one-fifth of the remaining candies is the same as multiplying by four-fifths.

  $1-\frac{1}{5}=\frac{4}{5}$

The following flow chart is more useful.



Step 2: Use a reverse flow chart to work backwards.



Ruth had 30 candies at the beginning.

Simplifying a Problem

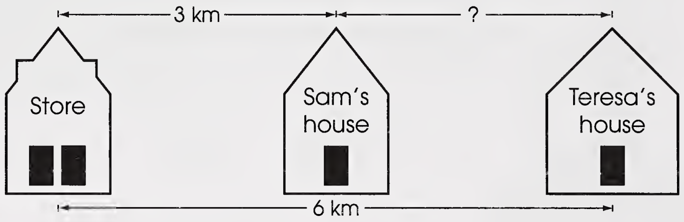
Are you sometimes confused about which operations to perform in a problem because the numbers in the problem are very large or because the problem seems very complicated? There are strategies you can use to simplify a problem.

Example

The average distance from the Sun to Mars is 228 000 000 km, and the average distance from the Sun to Earth is 150 000 000 km. What is the average distance between Earth and Mars?

Solution

Step 1: If you find the large numbers overwhelming, make up a related but simpler problem.

> Sam and Teresa both live east of the store. Sam lives 3 km from the store and Teresa lives 6 km from the store. How far is it from Sam's house to Teresa's house?
>
> 
>
>
> $6 - 3 = 3$
>
> It is 3 km from Sam's house to Teresa's house.

Step 2: Once you understand the related but simpler problem, you can solve the original problem.

$$228\ 000\ 000 - 150\ 000\ 000 = 78\ 000\ 000$$

The average distance from Earth to Mars is 78 000 000 km.

Finding and Applying a Pattern

To solve some problems, it is often helpful to use a related but simpler problem to discover a pattern.

Example

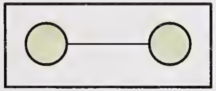
There are 20 students in a room. If every student shakes hands with every other student in the room, how many handshakes are exchanged?

Solution

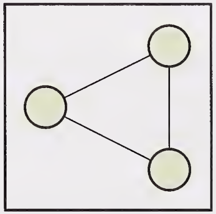
Step 1: Try solving the problem with 1, 2, 3, 4, or 5 students.

- If there is 1 student, no handshakes can be exchanged (as illustrated in the following diagram).

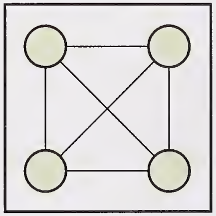
- If there are 2 students, 1 handshake can be exchanged (as illustrated in the following diagram).



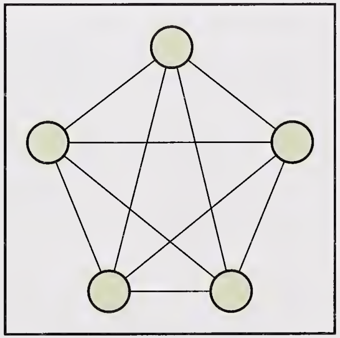
- If there are 3 students, 3 handshakes can be exchanged (as illustrated in the following diagram).



- If there are 4 students, 6 handshakes can be exchanged (as illustrated in the following diagram).



- If there are 5 students, 10 handshakes can be exchanged (as illustrated in the following diagram).



Step 2: Look for a pattern.

People	Handshakes	Pattern
1	0	
2	1	+1
3	3	+2
4	6	+3
5	10	+4

Step 3: Apply the pattern.

A calculator is helpful for applying the pattern. Go up to 19, because each student will shake the hand of 19 others.

1 + 2 + 3 + 4 + 5 + 6 +
7 + 8 + 9 + 10 + 11 +
12 + 13 + 14 + 15 +
16 + 17 + 18 + 19 =

190.

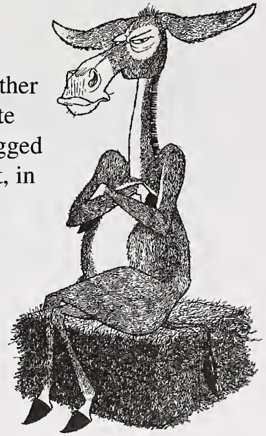
There are a total of 190 handshakes exchanged.

Using Elimination

Detectives are skilful at eliminating clues and solving mysteries. You can become skilful at using elimination to solve problems too.

Example

Five animals had a race over a short distance. Neither the donkey nor the coyote beat the lion. The coyote beat the rabbit, but not the ostrich. The donkey lagged behind the rabbit. If the ostrich was not the fastest, in what order did the animals finish the race?



Solution

Step 1: Decide what clues are given. The clues indicate the following:

- The donkey and the coyote did not finish first because they did not beat the lion.
- The rabbit did not finish first because the coyote beat it.
- The ostrich was faster than the coyote.
- The rabbit was faster than the donkey.
- The ostrich did not finish first because it was not the fastest.

The clues can be shown on a table.

	Donkey	Coyote	Rabbit	Ostrich	Lion
First	✗	✗	✗	✗	
Second					
Third					
Fourth					
Fifth					

Step 2: Use elimination and reconsider the clues.

By elimination, it is clear that the lion was first. Now, reconsider the clues:

- The rabbit did not finish second because the coyote beat it.
- The coyote did not finish second because the ostrich was faster.
- The donkey did not finish second because it lagged behind the rabbit.

The additional clues can be shown in the table.

	Donkey	Coyote	Rabbit	Ostrich	Lion
First	✗	✗	✗	✗	✓
Second	✗	✗	✗		✗
Third					✗
Fourth					✗
Fifth					✗

Step 3: Use elimination and reconsider the clues.

By elimination, it is clear that the ostrich finished second. Now, reconsider the clues:

- The rabbit did not finish third because the coyote beat it.
- The donkey did not finish third because it lagged behind the rabbit.

The additional clues can be shown in the table.

	Donkey	Coyote	Rabbit	Ostrich	Lion
First	✗	✗	✗	✗	✓
Second	✗	✗	✗	✓	✗
Third	✗		✗	✗	✗
Fourth				✗	✗
Fifth				✗	✗

Step 4: Use elimination and reconsider the clues.

By elimination, it is clear that the coyote finished third. Now, reconsider the clues.

Because the donkey lagged behind the rabbit, the rabbit was fourth and the donkey was fifth.

	Donkey	Coyote	Rabbit	Ostrich	Lion
First	✗	✗	✗	✗	✓
Second	✗	✗	✗	✓	✗
Third	✗	✓	✗	✗	✗
Fourth	✗	✗	✓	✗	✗
Fifth	✓	✗	✗	✗	✗

So, the lion was first, the ostrich was second, the coyote was third, the rabbit was fourth, and the donkey was last.

Using Truth Tables

A strategy you can use to solve some logic problems is making truth tables. Truth tables are similar to elimination tables.

Example

Detective Beagle is investigating a robbery in the forest. Charlie Chipmunk and Sammy Squirrel each deny being the robber. Harold Hare blames Charlie Chipmunk. Detective Beagle knows two of the animals are lying and one is telling the truth. Who is the robber?

Solution

Step 1: Assume that Charlie Chipmunk was the robber.

- Charlie Chipmunk said he didn't do it. If Charlie Chipmunk was the robber, this is a false statement. Make a truth table and put **F** under Charlie's name.

Charlie Chipmunk	Sammy Squirrel	Harold Hare
F		

- Sammy Squirrel denied doing it. If Charlie Chipmunk was the robber, this is a true statement. Put **T** under Sammy's name in the truth table.

Charlie Chipmunk	Sammy Squirrel	Harold Hare
F	**T**	

- Harold Hare blamed Charlie Chipmunk. If Charlie Chipmunk was the robber, this is a true statement. Put **T** under Harold's name in the truth table.

Charlie Chipmunk	Sammy Squirrel	Harold Hare
F	T	**T**

Detective Beagle knows two animals are lying. So, Charlie is not the robber.

Step 2: Assume that Sammy Squirrel was the robber.

- Charlie Chipmunk denied doing it. If Sammy Squirrel was the robber, this is a true statement. Put **T** under Charlie's name in the truth table.

Charlie Chipmunk	Sammy Squirrel	Harold Hare
F	T	T
T		

- Sammy Squirrel denied doing it. If Sammy Squirrel was the robber, this is a false statement. Put **F** under Sammy's name in the truth table.

Charlie Chipmunk	Sammy Squirrel	Harold Hare
F	T	T
T	**F**	

- Harold Hare blamed Charlie Chipmunk. If Sammy Squirrel was the robber, this is a false statement. Put **F** under Harold's name in the truth table.

Charlie Chipmunk	Sammy Squirrel	Harold Hare
F	T	T
T	F	**F**

Detective Beagle knows two of the animals are lying and one is telling the truth.

So, Sammy Squirrel was the robber.

Using an Equation

A strategy you can use to solve some problems is to write and solve an equation.

Example

Twelve less than a number is -3. Find the number.

Solution

Step 1: Let the unknown be equal to a variable. Let the number be n.

Step 2: Write an equation.

$$n - 12 = -3$$

Twelve less than a number is -3.

Step 3: Solve the equation.

$$\begin{array}{rcr} n - 12 & = & -3 \\ +12 & & +12 \\ \hline n & = & 9 \end{array}$$

The number is 9.

Example

The second of two numbers is 7 times the first. The sum of the two numbers is 32. Find the numbers.

Solution

Step 1: Write statements for the unknowns. Use only one variable.

Let n be the first number and let $7n$ be the second number.

The second of two numbers is 7 times the first.

Step 2: Write an equation.

$$n + 7n = 32$$

The sum of the two numbers is 32.

Step 3: Solve the equation.

$$\begin{aligned} n + 7n &= 32 \\ 8n &= 32 \\ \frac{8n}{8} &= \frac{32}{8} \\ n &= 4 \end{aligned}$$

The first number is 4.

Step 4: Find the second number if the first number is 4.

$$\begin{aligned} 7n &= 7 \times 4 \\ &= 28 \end{aligned}$$

The second number is 28.

So, the numbers are 4 and 28.

Example

Yvonne has 20 more nickels than dimes in her piggy bank. If the total value of the nickels and dimes is \$8.50, how many coins of each type does she have?

Solution

Step 1: Write statements for the unknowns. Use only one variable.

Let the number of dimes be n.

The value of the nickels, the number of dimes, and the value of the dimes must also be represented by algebraic expressions.

A table can be used to organize the information.

Type of Coin	Number of Coins	Value in Cents
Dimes	n	$10n$
Nickels	$n+20$	$5(n+20)$
Total		850

The value of the dimes is 10 times the number of dimes.

The value of the nickels is 5 times the number of nickels.

Yvonne has 20 more nickels than dimes.

This is given.

Step 2: Write an equation.

$$5(n+20)+10n=850$$

Step 3: Solve the equation.

$$\begin{aligned} 5(n+20)+10n &= 850 \\ 5n+100+10n &= 850 \\ 15n+100 &= 850 \\ -100 \quad & \quad -100 \\ \hline 15n \quad &= 750 \\ \frac{15n}{15} &= \frac{750}{15} \\ n &= 50 \end{aligned}$$

Yvonne has 50 dimes in her piggy bank.

Step 4: Find the number of nickels if the number of dimes is 50.

$$\begin{aligned} n+20 &= 50+20 \\ &= 70 \end{aligned}$$

Yvonne has 70 nickels in her piggy bank.

So, Yvonne has 50 dimes and 70 nickels in her piggy bank.

Glossary

adding from the left: a mental computation strategy in which you add columns of digits from left to right; also called the **left-to-right method**

algorithm: a set of steps for finding the answer to a problem

ascending order: in order from least to greatest

basic fraction: a fraction in simplest form

bivariate data: data that involves a relationship between two measures

cancelling: (in multiplication) the process of dividing a numerator and a denominator by a common factor

Cartesian coordinate system: a way of defining the position of a point in two-dimensional space

complex fraction: a fraction that has a fraction in the numerator and/or in the denominator

computer database: a collection of data organized for rapid search and retrieval

data: factual information that is gathered and used for calculations, comparisons, and discussion

denominator: the bottom number of a fraction; the divisor

descending order: in order from greatest to least

dividend: the number that is to be divided

divisor: the number that the dividend is divided by

ellipsis: a set of three dots indicating a continuing pattern

equivalent decimals: decimals that name the same part of a whole

equivalent fractions: fractions that represent the same part of a whole

extrapolate: to use information on a graph to estimate values that go beyond the graph

factor: a number that can be multiplied to obtain a given product

first term: (of a ratio) the first number in a ratio

front-end digits: an estimating method in which only the first digit or digits in each number is used and zeros are place holders

improper fraction: a fraction in which the numerator is greater than the denominator

interpolate: to read information directly from a graph

inverse operations: operations that undo each other

left-to-right method: a mental computation strategy in which you add columns of digits from left to right; also called **adding from the left**

line of best fit: a line drawn through the points of a scatter plot that best estimates the relationship between the two variables

lowest terms: (of a ratio) a ratio written in simplest form; also called **simplest form**

mixed number: a number expressed as a sum of a whole number and a proper fraction

multiplicative inverses: two numbers whose product is 1; also called reciprocals

e.g., The multiplicative inverse of 5 is $\frac{1}{5}$; the multiplicative inverse of $\frac{3}{4}$ is $\frac{4}{3}$.

numerator: the top number of a fraction; the dividend

order: list numbers in order of size

origin: on a graph, the point where the x-axis and y-axis meet

percent: a special ratio with 100 as the second term

problem: a task for which the method of finding the answer (as well as the answer) is not immediately known

product: the result of a multiplication

proper fraction: a fraction in which the numerator is less than the denominator

proportion: an equation showing that two ratios are equivalent

proportional ratios: equivalent ratios

quadrant: one of the four regions formed by the axes on a graph

quotient: the result of a division

rate: a comparison that describes a unit relationship as well as a number relationship

ratio: a comparison that describes a number relationship between quantities expressed in the same unit

repeating decimal number: a decimal number with an infinite number of non-zero digits and a repeating pattern

rounding: an estimating method in which a number is expressed to the nearest whole, nearest tenth, and so on

scatter plot: a graph of a set of points representing the relationship between two sets of numbers or data

second term: (of a ratio) the second number in a ratio

statistics: the branch of mathematics that deals with the systematic collection and organization of numerical information; a number of pieces of information which have been collected and recorded

terminating decimal number: a decimal number with a finite number of non-zero digits

Suggested Answers

Section 1: Activity 1

1. a. $\frac{7}{12}$ b. $\frac{19}{24}$

2. The fraction of cartons containing doughnuts is $\frac{17}{12}$ or $1\frac{5}{12}$.

3. a. $1 = \frac{\mathbf{10}}{\mathbf{10}}$ b. $\frac{2}{3} = \frac{\mathbf{4}}{\mathbf{6}}$

4. Answers may vary. Samples of equivalent fractions are given.

$$\frac{5}{12} = \frac{10}{24} = \frac{15}{36} = \frac{20}{48} = \frac{25}{60}$$

5. a. $\frac{9}{12} = \frac{3}{4}$ b. $\frac{4}{6} = \frac{2}{3}$ c. $\frac{24}{96} = \frac{1}{4}$

   d. $\frac{72}{30} = \frac{12}{5}$ e. $\frac{30}{24} = \frac{5}{4}$ f. $\frac{45}{40} = \frac{9}{8}$

6. a. $5\frac{1}{4} = \frac{21}{4}$ b. $3\frac{2}{5} = \frac{17}{5}$

   c. $2\frac{1}{3} = \frac{7}{3}$ d. $4\frac{3}{5} = \frac{23}{5}$

7. a. $\frac{10}{3} = 3\frac{1}{3}$ b. $\frac{16}{5} = 3\frac{1}{5}$

   c. $\frac{21}{4} = 5\frac{1}{4}$ d. $\frac{33}{2} = 16\frac{1}{2}$

8. a. $0.3 = \frac{3}{10}$ b. $0.26 = \frac{26}{100} = \frac{13}{50}$ c. $0.05 = \frac{5}{100} = \frac{1}{20}$

   d. $4.25 = 4\frac{25}{100} = 4\frac{1}{4}$ e. $2.875 = 2\frac{875}{1000} = 2\frac{7}{8}$ f. $3.036 = 3\frac{36}{1000} = 3\frac{9}{250}$

9. a. $\frac{3}{4} = \frac{75}{100} = 0.75$ b. $\frac{2}{5} = \frac{4}{10} = 0.4$

   c. $1\frac{3}{10} = 1.3$ d. $2\frac{1}{2} = 2\frac{5}{10} = 2.5$

10. a. $\frac{7}{9} = 0.7$ b. $\frac{5}{12} = 0.416$ c. $\frac{9}{16} = 0.5625$

    d. $\frac{9}{11} = 0.\overline{81}$ e. $\frac{3}{40} = 0.075$ f. $\frac{2}{13} = 0.\overline{153\,846}$

Note: The numbers in 10.c. and 10.e. are terminating decimals.

11. **a.** $\frac{1}{9}=0.\overline{1}$ $\quad \frac{2}{9}=0.\overline{2}$ $\quad \frac{3}{9}=0.\overline{3}$ $\quad \frac{4}{9}=0.\overline{4}$

b. $\underset{1\times 0.\overline{1}}{\underset{\uparrow}{0.\overline{1}}},\ \underset{2\times 0.\overline{1}}{\underset{\uparrow}{0.\overline{2}}},\ \underset{3\times 0.\overline{1}}{\underset{\uparrow}{0.\overline{3}}},\ \underset{4\times 0.\overline{1}}{\underset{\uparrow}{0.\overline{4}}}$

In this case, the repeating decimal relates to the numerator of each fraction.

c. Using the pattern, you should have predicted the following:

$\frac{5}{9}=\underset{5\times 0.\overline{1}}{\underset{\uparrow}{0.\overline{5}}}$ $\quad \frac{6}{9}=\underset{6\times 0.\overline{1}}{\underset{\uparrow}{0.\overline{6}}}$ $\quad \frac{7}{9}=\underset{7\times 0.\overline{1}}{\underset{\uparrow}{0.\overline{7}}}$ $\quad \frac{8}{9}=\underset{8\times 0.\overline{1}}{\underset{\uparrow}{0.\overline{8}}}$

12. **a.** $\frac{1}{99}=0.\overline{01}$ $\quad \frac{2}{99}=0.\overline{02}$ $\quad \frac{3}{99}=0.\overline{03}$ $\quad \frac{4}{99}=0.\overline{04}$ $\quad \frac{5}{99}=0.\overline{05}$

b. $\underset{1\times 0.\overline{01}}{\underset{\uparrow}{0.\overline{01}}}$ $\quad \underset{2\times 0.\overline{01}}{\underset{\uparrow}{0.\overline{02}}}$ $\quad \underset{3\times 0.\overline{01}}{\underset{\uparrow}{0.\overline{03}}}$ $\quad \underset{4\times 0.\overline{01}}{\underset{\uparrow}{0.\overline{04}}}$ $\quad \underset{5\times 0.\overline{01}}{\underset{\uparrow}{0.\overline{05}}}$

The pattern relates directly to the numerator.

c. Using the pattern, you should have predicted the following:

$\frac{13}{99}=\underset{13\times 0.\overline{01}}{\underset{\uparrow}{0.\overline{13}}}$ $\quad \frac{23}{99}=\underset{23\times 0.\overline{01}}{\underset{\uparrow}{0.\overline{23}}}$ $\quad \frac{47}{99}=\underset{47\times 0.\overline{01}}{\underset{\uparrow}{0.\overline{47}}}$ $\quad \frac{68}{99}=\underset{68\times 0.\overline{01}}{\underset{\uparrow}{0.\overline{68}}}$ $\quad \frac{94}{99}=\underset{94\times 0.\overline{01}}{\underset{\uparrow}{0.\overline{94}}}$

13. **a.** $2.\overline{7}=2\frac{7}{9}$ $\qquad$ **b.** $0.\overline{14}=\frac{14}{99}$

c. $3.\overline{26}=3\frac{26}{99}$ $\qquad$ **d.** $1.\overline{8}=1\frac{8}{9}$

14. **Method 1: Using Long Division**

$$
\begin{array}{r}
12.5 \\
36\overline{)450.0} \\
\underline{36} \\
90 \\
\underline{72} \\
18\,0 \\
\underline{18\,0} \\
0
\end{array}
$$

The price of one ticket was $12.50.

Method 2: Using a Calculator

4 5 0 ÷ 3 6 =

12.5

The price of one ticket was $12.50.

15. Method 1: Using Long Division

$$12\overline{)4000} = 333\text{ R}4 = 333\tfrac{4}{12} = 333\tfrac{1}{3}$$

$$\begin{array}{r} 333\text{ R}4 \\ 12\overline{)4000} \\ \underline{36} \\ 40 \\ \underline{36} \\ 40 \\ \underline{36} \\ 4 \end{array}$$

The Egg Marketing Board receives $333\frac{1}{3}$ dozen eggs.

Method 2: Using a Calculator

4 0 0 0 ÷ 1 2 ENTER

333.3333333

The Egg Marketing Board receives $333.\overline{3}$ dozen eggs.

16. a.

$$\begin{array}{r} 55\text{ R}15 \\ 24\overline{)1335} \\ \underline{120} \\ 135 \\ \underline{120} \\ 15 \end{array}$$

1 3 3 5 ÷ 2 4 =

55.625

The teens needed 56 boxes.

b. There were 55 full boxes.

c. There were 15 soda cans in the partly filled box.

Note: If you used a calculator for question 16, you could have found the answer to 16.c. by pressing these keys.

1 3 3 5 ÷ 2 4 =

55.625 ← Find the quotient.

− 5 5 =

0.625 ← Subtract the whole number part of the quotient.

× 2 4 =

15. ← This is the remainder.

17. a. $\frac{1}{6} < \frac{5}{6}$ **b.** $\frac{3}{4} > \frac{1}{4}$ **c.** $\frac{5}{12} < \frac{7}{12}$

18. a. $\frac{1}{6} < \frac{1}{2}$ **b.** $\frac{1}{2} > \frac{1}{3}$

c. $\frac{5}{6} > \frac{5}{12}$ **d.** $\frac{7}{4} > \frac{7}{12}$

19. a. $\frac{2}{5}=\frac{14}{35}$ $\quad \frac{3}{7}=\frac{15}{35}$

$\frac{14}{35}<\frac{15}{35}$

$\therefore \frac{2}{5} < \frac{3}{7}$

b. $\frac{7}{9}$ $\quad \frac{2}{3}=\frac{6}{9}$

$\frac{7}{9}>\frac{6}{9}$

$\therefore \frac{7}{9} > \frac{2}{3}$

c. $\frac{3}{4}=\frac{15}{20}$ $\quad \frac{4}{5}=\frac{16}{20}$

$\frac{15}{20}<\frac{16}{20}$

$\therefore \frac{3}{4} < \frac{4}{5}$

d. $\frac{5}{6}=\frac{10}{12}$ $\quad \frac{11}{12}$

$\frac{10}{12}<\frac{11}{12}$

$\therefore \frac{5}{6} < \frac{11}{12}$

20. $\frac{1}{2}=\frac{2}{4}$ $\quad \frac{3}{4}$

$\frac{2}{4}<\frac{3}{4}$

$\therefore \frac{1}{2}<\frac{3}{4}$

Darwin took more time to complete the work.

21. $\frac{1}{2}=\frac{12}{24}$ $\quad \frac{5}{8}=\frac{15}{24}$ $\quad \frac{2}{3}=\frac{16}{24}$

$\frac{12}{24}<\frac{15}{24}<\frac{16}{24}$

$\therefore \frac{1}{2}<\frac{5}{8}<\frac{2}{3}$

The biggest diamond of the three is the $\frac{2}{3}$-carat diamond.

22. a. $\frac{7}{8}=0.875$ and $0.875>0.75$

$\therefore \frac{7}{8} > 0.75$

b. $\frac{5}{8}=0.625$ and $0.625>0.6$

$\therefore \frac{5}{8} > 0.6$

c. $0.6=\frac{6}{10}$ $\quad \frac{2}{3}=\frac{20}{30}$

$=\frac{18}{30}$

$\frac{18}{30}<\frac{20}{30}$

$\therefore 0.6 < \frac{2}{3}$

d. $\frac{1}{7}=\frac{10}{70}$ $\quad 0.2=\frac{2}{10}$

$=\frac{14}{70}$

$\frac{10}{70}<\frac{14}{70}$

$\therefore \frac{1}{7} < 0.2$

Looking Back

23. **a.** An improper fraction is a fraction in which the numerator is larger than the denominator. There is an infinite number of improper fractions; one example is $\frac{15}{2}$.

b. A mixed number consists of a whole number and a fraction. You probably use mixed numbers more often than improper fractions in everyday language. One example is $1\frac{1}{2}$.

24. "I'll get time-and-a-half if I work on the holiday."

Time-and-a-half means you will be paid $1\frac{1}{2}$ times your regular hourly wage. For example, if your hourly wage is \$8/h, you will get the \$8 plus an additional $\frac{1}{2}(8) = \$4$ for every hour you work. The total amount per hour will be \$12.

"They scored the most points in the first quarter."

The first quarter means the game time is split into four equal time periods, with the most points being scored in the first time period.

Section 1: Activity 2

1. $\frac{3}{8}+\frac{1}{4}=\frac{3}{8}+\frac{2}{8}$
$=\frac{5}{8}$

or

$$\begin{array}{r} \frac{3}{8} \\ +\ \frac{1}{4} \\ \hline \end{array} \Rightarrow \begin{array}{r} \frac{3}{8} \\ +\ \frac{2}{8} \\ \hline \frac{5}{8} \end{array}$$

Mark spent $\frac{5}{8}$ of the day on these two activities.

2. $\frac{3}{10}+\frac{5}{10}=\frac{8}{10}$
$=\frac{4}{5}$

or

$$\begin{array}{r} \frac{3}{10} \\ +\ \frac{5}{10} \\ \hline \frac{8}{10}=\frac{4}{5} \end{array}$$

Françoise gave away $\frac{4}{5}$ of her jewellery collection.

3. $\frac{1}{2}+\frac{1}{4}=\frac{2}{4}+\frac{1}{4}$
$=\frac{3}{4}$

or

$$\begin{array}{r} \frac{1}{2} \\ +\ \frac{1}{4} \\ \hline \end{array} \Rightarrow \begin{array}{r} \frac{2}{4} \\ +\ \frac{1}{4} \\ \hline \frac{3}{4} \end{array}$$

Lucy and Ruth scored $\frac{3}{4}$ of the goals altogether.

4. $\frac{1}{3}+\frac{1}{4}=\frac{4}{12}+\frac{3}{12}$
$=\frac{7}{12}$

or

$$\begin{array}{r} \frac{1}{3} \\ +\ \frac{1}{4} \\ \hline \end{array} \Rightarrow \begin{array}{r} \frac{4}{12} \\ +\ \frac{3}{12} \\ \hline \frac{7}{12} \end{array}$$

Katrina used $\frac{7}{12}$ of a can of varnish altogether.

5. $\frac{7}{8}-\frac{1}{8}=\frac{6}{8}$
$=\frac{3}{4}$

or

$$\begin{array}{r} \frac{7}{8} \\ -\ \frac{1}{8} \\ \hline \frac{6}{8}=\frac{3}{4} \end{array}$$

Mr. Yakimchuk used $\frac{3}{4}$ of a tank of gas.

6. $\frac{3}{4}-\frac{1}{4}=\frac{2}{4}$
$=\frac{1}{2}$

or

$$\begin{array}{r} \frac{3}{4} \\ -\ \frac{1}{4} \\ \hline \frac{2}{4}=\frac{1}{2} \end{array}$$

It took Lori $\frac{1}{2}$ h longer.

7. $1-\frac{1}{4}=\frac{4}{4}-\frac{1}{4}$
$=\frac{3}{4}$

or

$$\begin{array}{r} 1 \\ -\ \frac{1}{4} \\ \hline \end{array} \Rightarrow \begin{array}{r} \frac{4}{4} \\ -\ \frac{1}{4} \\ \hline \frac{3}{4} \end{array}$$

Harpreet left $\frac{3}{4}$ of his garden for other plants.

8. $\frac{1}{2}-\frac{1}{3}=\frac{3}{6}-\frac{2}{6}$
$=\frac{1}{6}$

or

$$\begin{array}{r} \frac{1}{2} \\ -\ \frac{1}{3} \\ \hline \end{array} \Rightarrow \begin{array}{r} \frac{3}{6} \\ -\ \frac{2}{6} \\ \hline \frac{1}{6} \end{array}$$

Jana ate $\frac{1}{6}$ of a bag of popcorn on the bus.

9. a. $\frac{1}{5}+\frac{2}{5}=\frac{3}{5}$

b. $\frac{1}{10}+\frac{6}{10}=\frac{7}{10}$

c. $\frac{1}{6}+\frac{4}{6}=\frac{5}{6}$

d. $\frac{1}{8}+\frac{1}{2}=\frac{5}{8}$

e. $\frac{2}{3}+\frac{1}{6}=\frac{5}{6}$

f. $\frac{2}{5}+\frac{3}{10}=\frac{7}{10}$

g. $$\begin{array}{r} \frac{3}{4} \\ +\ \frac{1}{8} \\ \hline \frac{7}{8} \end{array}$$

h. $$\begin{array}{r} \frac{1}{2} \\ +\ \frac{3}{8} \\ \hline \frac{7}{8} \end{array}$$

i. $$\begin{array}{r} \frac{5}{6} \\ +\ \frac{1}{12} \\ \hline \frac{11}{12} \end{array}$$

10. a. $\frac{7}{8}-\frac{1}{8}=\frac{6}{8}$

Note: You are usually expected to give a fraction answer in its simplest form.

$$\frac{6}{8}=\frac{3}{4}$$

b. $\frac{3}{4}-\frac{1}{2}=\frac{1}{4}$

c. $\frac{1}{2}-\frac{1}{4}=\frac{1}{4}$

d. $$\begin{array}{r} \frac{2}{5} \\ -\ \frac{1}{10} \\ \hline \frac{3}{10} \end{array}$$

e. $$\begin{array}{r} \frac{7}{12} \\ -\ \frac{1}{6} \\ \hline \frac{5}{12} \end{array}$$

f. $$\begin{array}{r} \frac{5}{6} \\ -\ \frac{2}{3} \\ \hline \frac{1}{6} \end{array}$$

11. a.
$$\begin{aligned} \frac{1}{2}+\frac{2}{5}+\frac{3}{5}+\frac{3}{4} &= \left(\frac{2}{5}+\frac{3}{5}\right)+\left(\frac{1}{2}+\frac{3}{4}\right) \\ &= \frac{5}{5}+\left(\frac{2}{4}+\frac{3}{4}\right) \\ &= 1+\frac{5}{4} \\ &= 1+1\frac{1}{4} \\ &= 2\frac{1}{4} \end{aligned}$$

b.
$$\begin{aligned} \frac{1}{3}+\frac{1}{6}+\frac{1}{3}+\frac{1}{2} &= \left(\frac{1}{3}+\frac{1}{3}\right)+\left(\frac{1}{6}+\frac{1}{2}\right) \\ &= \frac{2}{3}+\left(\frac{1}{6}+\frac{3}{6}\right) \\ &= \frac{2}{3}+\frac{4}{6} \\ &= \frac{2}{3}+\frac{2}{3} \\ &= \frac{4}{3} \\ &= 1\frac{1}{3} \end{aligned}$$

c.
$$\begin{aligned} \frac{1}{2}+\frac{3}{4}+\frac{1}{2}+\frac{7}{8} &= \left(\frac{1}{2}+\frac{1}{2}\right)+\left(\frac{3}{4}+\frac{7}{8}\right) \\ &= \frac{2}{2}+\left(\frac{6}{8}+\frac{7}{8}\right) \\ &= 1+\frac{13}{8} \\ &= 1+1\frac{5}{8} \\ &= 2\frac{5}{8} \end{aligned}$$

d.
$$\begin{aligned} \frac{2}{5}+\frac{1}{5}+\frac{3}{10}+\frac{2}{5} &= \left(\frac{2}{5}+\frac{1}{5}+\frac{2}{5}\right)+\frac{3}{10} \\ &= \frac{5}{5}+\frac{3}{10} \\ &= 1\frac{3}{10} \end{aligned}$$

12. a.

$$\begin{array}{r} 3\frac{1}{10} \\ +\ 1\frac{3}{5} \\ \hline \end{array} \Rightarrow \begin{array}{r} 3\frac{1}{10} \\ +\ 1\frac{6}{10} \\ \hline 4\frac{7}{10} \end{array}$$

b.

$$\begin{array}{r} 4\frac{1}{6} \\ +\ 1\frac{2}{3} \\ \hline \end{array} \Rightarrow \begin{array}{r} 4\frac{1}{6} \\ +\ 1\frac{4}{6} \\ \hline 5\frac{5}{6} \end{array}$$

c.

$$\begin{array}{r} 5\frac{5}{6} \\ +\ 1\frac{5}{12} \\ \hline \end{array} \Rightarrow \begin{array}{r} 5\frac{10}{12} \\ +\ 1\frac{5}{12} \\ \hline \frac{15}{12} \end{array} \Rightarrow \begin{array}{r} \overset{1}{5}\frac{10}{12} \\ +\ 1\frac{5}{12} \\ \hline 7\frac{3}{12} = 7\frac{1}{4} \end{array}$$

This step can be done mentally.

d.

$$\begin{array}{r} 1\frac{3}{4} \\ +\ 1\frac{1}{2} \\ \hline \end{array} \Rightarrow \begin{array}{r} 1\frac{3}{4} \\ +\ 1\frac{2}{4} \\ \hline \frac{5}{4} \end{array} \Rightarrow \begin{array}{r} \overset{1}{1}\frac{3}{4} \\ +\ 1\frac{2}{4} \\ \hline 3\frac{1}{4} \end{array}$$

This step can be done mentally.

13.

$$\begin{array}{r} 10\frac{1}{2} \\ +\ 8\frac{1}{3} \\ \hline \end{array} \Rightarrow \begin{array}{r} 10\frac{3}{6} \\ +\ 8\frac{2}{6} \\ \hline 18\frac{5}{6} \end{array}$$

The two charities collected $\$18\frac{5}{6}$ million altogether.

14. a.

$$\begin{aligned} 1\frac{7}{8}+2\frac{3}{4} &= \frac{15}{8}+\frac{11}{4} \\ &= \frac{15}{8}+\frac{22}{8} \\ &= \frac{37}{8} \\ &= 4\frac{5}{8} \end{aligned}$$

b.

$$\begin{aligned} 1\frac{1}{5}+2\frac{3}{10} &= \frac{6}{5}+\frac{23}{10} \\ &= \frac{12}{10}+\frac{23}{10} \\ &= \frac{35}{10} \\ &= \frac{7}{2} \\ &= 3\frac{1}{2} \end{aligned}$$

c.

$$\begin{aligned} 1\frac{2}{3}+3\frac{3}{4} &= \frac{5}{3}+\frac{15}{4} \\ &= \frac{20}{12}+\frac{45}{12} \\ &= \frac{65}{12} \\ &= 5\frac{5}{12} \end{aligned}$$

15. $1\frac{3}{4}+1\frac{1}{2}=\frac{7}{4}+\frac{3}{2}$

$=\frac{7}{4}+\frac{6}{4}$

$=\frac{13}{4}$

$=3\frac{1}{4}$

Mrs. Crowell transplanted $3\frac{1}{4}$ dozen tulips.

16.

$$\begin{array}{r} 1\frac{1}{2} \\ +\ 2\frac{3}{4} \\ \hline \end{array} \Rightarrow \begin{array}{r} 1\ \frac{2}{4} \\ +\ 2\ \frac{3}{4} \\ \hline \frac{5}{4} \end{array} \Rightarrow \begin{array}{r} \overset{1}{1}\frac{2}{4} \\ +\ 2\frac{3}{4} \\ \hline 4\frac{1}{4} \end{array}$$

This step can be done mentally.

Altogether, Franscesca spent $4\frac{1}{4}$ h working in the greenhouse on these two days.

17. a.

$$\begin{array}{r} 3\frac{1}{2} \\ -\ 1\frac{1}{4} \\ \hline \end{array} \Rightarrow \begin{array}{r} 3\frac{2}{4} \\ -\ 1\frac{1}{4} \\ \hline 2\frac{1}{4} \end{array}$$

b.

$$\begin{array}{r} 4\frac{3}{4} \\ -\ 1\frac{1}{2} \\ \hline \end{array} \Rightarrow \begin{array}{r} 4\frac{3}{4} \\ -\ 1\frac{2}{4} \\ \hline 3\frac{1}{4} \end{array}$$

c.

$$\begin{array}{r} 5\frac{1}{8} \\ -\ 2\frac{3}{4} \\ \hline \end{array} \Rightarrow \begin{array}{r} 5\frac{1}{8} \\ -\ 2\frac{6}{8} \\ \hline \end{array} \Rightarrow \begin{array}{r} 4\frac{9}{8} \\ -\ 2\frac{6}{8} \\ \hline 2\frac{3}{8} \end{array}$$

d.

$$\begin{array}{r} 6\frac{1}{4} \\ -\ 3\frac{5}{8} \\ \hline \end{array} \Rightarrow \begin{array}{r} 6\frac{2}{8} \\ -\ 3\frac{5}{8} \\ \hline \end{array} \Rightarrow \begin{array}{r} 5\frac{10}{8} \\ -\ 3\frac{5}{8} \\ \hline 2\frac{5}{8} \end{array}$$

18.

$$\begin{array}{r} 9\frac{1}{2} \\ -\ 8\frac{3}{4} \\ \hline \end{array} \Rightarrow \begin{array}{r} 9\frac{2}{4} \\ -\ 8\frac{3}{4} \\ \hline \end{array} \Rightarrow \begin{array}{r} 8\frac{6}{4} \\ -\ 8\frac{3}{4} \\ \hline \frac{3}{4} \end{array}$$

Tiffany worked $\frac{3}{4}$ h longer than Gerhardt.

19. a. $5\frac{7}{8}-2\frac{3}{4}=\frac{47}{8}-\frac{11}{4}$

$=\frac{47}{8}-\frac{22}{8}$

$=\frac{25}{8}$

$=3\frac{1}{8}$

b. $4\frac{2}{5}-2\frac{7}{10}=\frac{22}{5}-\frac{27}{10}$

$=\frac{44}{10}-\frac{27}{10}$

$=\frac{17}{10}$

$=1\frac{7}{10}$

c. $5\frac{1}{6} - 3\frac{7}{12} = \frac{31}{6} - \frac{43}{12}$

$= \frac{62}{12} - \frac{43}{12}$

$= \frac{19}{12}$

$= 1\frac{7}{12}$

20. $4 - 1\frac{1}{2} = 4 - \frac{3}{2}$

$= \frac{8}{2} - \frac{3}{2}$

$= \frac{5}{2}$

$= 2\frac{1}{2}$

George must still paint $2\frac{1}{2}$ chairs.

Looking Back

21. a. $\frac{1}{4} + \frac{1}{6}$

b. In the second diagram, the pieces are not the same size. You can only add pieces that are the same size.

c. It is always possible to find a common denominator. If you cannot determine the lowest common denominator, simply multiply the denominators to find a common denominator.

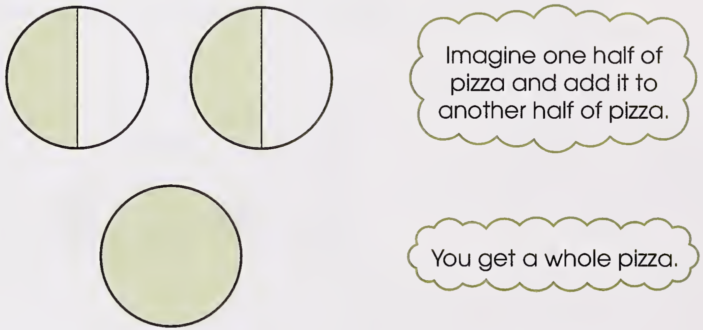
d. **Method 1**

$\frac{1}{2} + \frac{1}{2} = \frac{1+1}{2}$

$= \frac{2}{2}$

$= 1$

Method 2



Section 1: Activity 3

1. a. The numerator of each answer (before it is simplified) is the product of the numerators of the factors.

b. The denominator of each answer (before it is simplified) is the product of the denominators of the factors.

2. $\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$

Juanita mowed $\frac{3}{8}$ of the entire lawn in the afternoon.

3. $\frac{1}{3}\times\frac{3}{4}=\frac{3}{12}$

$=\frac{1}{4}$

It takes Dale $\frac{1}{4}$ h to ride his bike from home to the store.

4. $\frac{3}{5}\times\frac{2}{3}=\frac{6}{15}$

$=\frac{2}{5}$

Of the customers, $\frac{2}{5}$ had both soup and salad.

5. a. $\frac{1}{3}\times\frac{1}{2}=\frac{1}{6}$ b. $\frac{3}{4}\times\frac{3}{5}=\frac{9}{20}$ c. $\frac{2}{3}\times\frac{4}{5}=\frac{8}{15}$

6. a. $\frac{7}{8}\times\frac{4}{5}\times\frac{5}{8}=\frac{7\times\overset{1}{\cancel{4}}\times\overset{1}{\cancel{5}}}{8\times\underset{1}{\cancel{5}}\times\underset{2}{\cancel{8}}}$

$=\frac{7}{16}$

$\frac{5}{5}=\frac{1}{1};\ \frac{4}{8}=\frac{1}{2}$

or

$\frac{7}{8}\times\frac{4}{5}\times\frac{5}{8}=\frac{7\times\overset{1}{\cancel{4}}\times\overset{1}{\cancel{5}}}{\underset{2}{\cancel{8}}\times\underset{1}{\cancel{5}}\times 8}$

$=\frac{7}{16}$

$\frac{5}{5}=\frac{1}{1};\ \frac{4}{8}=\frac{1}{2}$

b. $\frac{1}{4}\times\frac{3}{5}\times\frac{2}{3}=\frac{1\times\overset{1}{\cancel{3}}\times\overset{1}{\cancel{2}}}{\underset{2}{\cancel{4}}\times 5\times\underset{1}{\cancel{3}}}$

$=\frac{1}{10}$

$\frac{3}{3}=\frac{1}{1};\ \frac{2}{4}=\frac{1}{2}$

c. $\frac{3}{5}\times\frac{4}{9}\times\frac{5}{6}=\frac{\overset{1}{\cancel{3}}\times\overset{2}{\cancel{4}}\times\overset{1}{\cancel{5}}}{\underset{1}{\cancel{5}}\times\underset{3}{\cancel{9}}\times\underset{3}{\cancel{6}}}$

$=\frac{2}{9}$

$\frac{3}{9}=\frac{1}{3};\ \frac{4}{6}=\frac{2}{3};\ \frac{5}{5}=\frac{1}{1}$

or

$\frac{3}{5}\times\frac{4}{9}\times\frac{5}{6}=\frac{\overset{1}{\cancel{3}}\times 4\times\overset{1}{\cancel{5}}}{\underset{1}{\cancel{5}}\times 9\times\underset{2}{\cancel{6}}}$

$=\frac{4}{18}$

$=\frac{2}{9}$

$\frac{3}{6}=\frac{1}{2};\ \frac{5}{5}=\frac{1}{1}$

d. $\frac{2}{3}\times\frac{5}{6}\times\frac{9}{10}=\frac{\overset{1}{\cancel{2}}\times\overset{1}{\cancel{5}}\times\overset{3}{\cancel{9}}}{\underset{1}{\cancel{3}}\times\underset{3}{\cancel{6}}\times\underset{2}{\cancel{10}}}$

$=\frac{3}{6}$

$=\frac{1}{2}$

$\frac{2}{6}=\frac{1}{3};\ \frac{5}{10}=\frac{1}{2};\ \frac{9}{3}=\frac{3}{1}$

or

$$\frac{2}{3}\times\frac{5}{6}\times\frac{9}{10}=\frac{2\times\cancel{5}^{1}\times\cancel{9}^{3}}{3\times\cancel{6}_{2}\times\cancel{10}_{2}}$$

$$\frac{5}{10}=\frac{1}{2};\ \frac{9}{6}=\frac{3}{2}$$

$$=\frac{6}{12}$$

$$=\frac{1}{2}$$

7. a. $\frac{1}{\cancel{2}_{1}}\times\frac{\cancel{2}^{1}}{3}=\frac{1}{3}$

b. $\frac{1}{\cancel{2}_{1}}\times\frac{\cancel{2}^{1}}{\cancel{3}_{1}}\times\frac{\cancel{3}^{1}}{4}=\frac{1}{4}$

c. $\frac{1}{\cancel{2}_{1}}\times\frac{\cancel{2}^{1}}{\cancel{3}_{1}}\times\frac{\cancel{3}^{1}}{\cancel{4}_{1}}\times\frac{\cancel{4}^{1}}{5}=\frac{1}{5}$

8. Each answer has the numerator of the first factor and the denominator of the last factor. Other numerators and denominators cancel to equal 1.

9. $\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}\times\frac{4}{5}\times\ldots\times\frac{49}{50}=\frac{1}{50}$

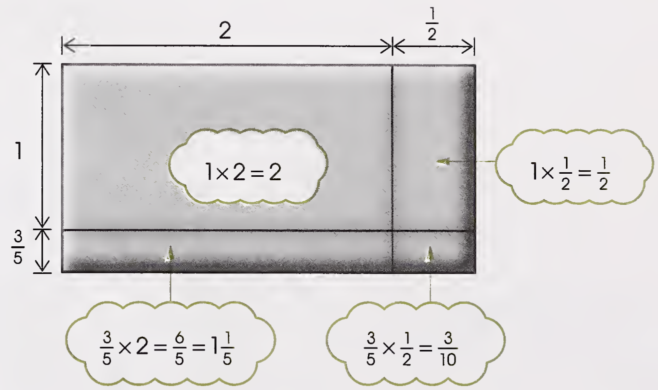
10. a. $1\frac{3}{5}\times2\frac{1}{2}=\frac{8}{5}\times\frac{5}{2}$

$$=\frac{\cancel{8}^{4}\times\cancel{5}^{1}}{\cancel{5}_{1}\times\cancel{2}_{1}}$$

$$=\frac{4}{1}$$

$$=4$$

The following rectangle can be used to illustrate the product.



Ask yourself, "What is the total area of the four sections of the rectangle?"

$$2+1\frac{1}{5}+\frac{1}{2}+\frac{3}{10}=2+1\frac{2}{10}+\frac{5}{10}+\frac{3}{10}$$

$$=3\frac{10}{10}$$

$$=4$$

$$\therefore\ 1\frac{3}{5}\times2\frac{1}{2}=4$$

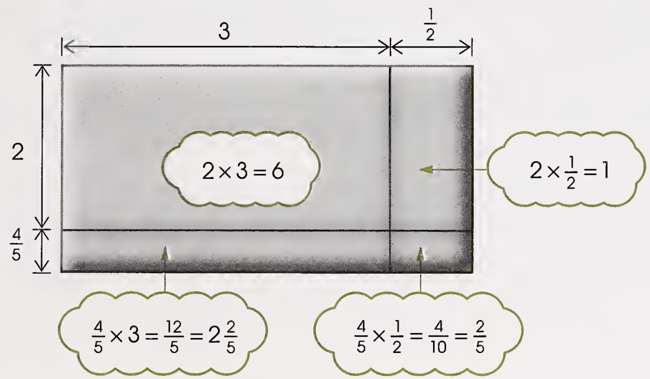
b. $2\frac{4}{5}\times 3\frac{1}{2}=\frac{14}{5}\times\frac{7}{2}$

$=\frac{\overset{7}{\cancel{14}}\times 7}{5\times\underset{1}{\cancel{2}}}$

$=\frac{49}{5}$

$=9\frac{4}{5}$

The following rectangle can be used to illustrate the product.



Ask yourself, "What is the total area of the four sections of the rectangle?"

$6+2\frac{2}{5}+1+\frac{2}{5}=9\frac{4}{5}$

$\therefore\ 2\frac{4}{5}\times 3\frac{1}{2}=9\frac{4}{5}$

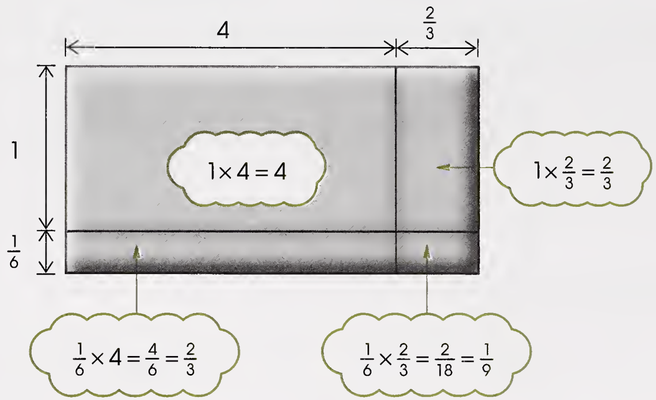
c. $1\frac{1}{6}\times 4\frac{2}{3}=\frac{7}{6}\times\frac{14}{3}$

$=\frac{7\times\overset{7}{\cancel{14}}}{\underset{3}{\cancel{6}}\times 3}$

$=\frac{49}{9}$

$=5\frac{4}{9}$

The following rectangle can be used to illustrate the product.



Ask yourself, "What is the total area of the four sections of the rectangle?"

$$4+\frac{2}{3}+\frac{2}{3}+\frac{1}{9}=4+\frac{6}{9}+\frac{6}{9}+\frac{1}{9}$$
$$=4+\frac{13}{9}$$
$$=4+1\frac{4}{9}$$
$$=5\frac{4}{9}$$

$$\therefore\ 1\frac{1}{6}\times 4\frac{2}{3}=5\frac{4}{9}$$

11. **a.** $2\times 4\frac{1}{2}=2\times\frac{9}{2}$

$$=\frac{\cancel{2}^{1}\times 9}{\cancel{2}_{1}}$$
$$=9$$

For twice the recipe, 9 cups of carrots are needed.

b. $\frac{1}{2}\times 4\frac{1}{2}=\frac{1}{2}\times\frac{9}{2}$

$$=\frac{9}{4}$$
$$=2\frac{1}{4}$$

For half the recipe, $2\frac{1}{4}$ cups of carrots are needed.

Looking Back

12. **a.** The quantities required for each ingredient are as follows:

Eggs

$$1\times\frac{3}{2}=\frac{3}{2}$$
$$=1\frac{1}{2}\text{ eggs}$$

Milk

$$1\frac{1}{4}\times\frac{3}{2}=\frac{5}{4}\times\frac{3}{2}$$
$$=\frac{15}{8}$$
$$=1\frac{7}{8}\text{ cups of milk}$$

Flour

$$1\frac{1}{2}\times\frac{3}{2}=\frac{3}{2}\times\frac{3}{2}$$
$$=\frac{9}{4}$$
$$=2\frac{1}{4}\text{ cups of flour}$$

Sugar

$$2\times\frac{3}{2}=\frac{\cancel{2}^{1}\times 3}{\cancel{2}_{1}}$$
$$=\frac{3}{1}$$
$$=3\text{ tbsp of sugar}$$

Shortening

$$3\times\frac{3}{2}=\frac{3\times 3}{2}$$
$$=\frac{9}{2}$$
$$=4\frac{1}{2}\text{ tbsp of shortening}$$

Salt

$$\frac{1}{2}\times\frac{3}{2}=\frac{1}{2}\times\frac{3}{2}$$
$$=\frac{3}{4}\text{ tbsp of salt}$$

Baking Powder

$$3\frac{1}{2}\times\frac{3}{2}=\frac{7}{2}\times\frac{3}{2}$$
$$=\frac{21}{4}$$
$$=5\frac{1}{4}\text{ tbsp of baking powder}$$

b. $\frac{15}{4}=3\frac{3}{4}$

You would multiply each ingredient by $\frac{15}{4}$ or $3\frac{3}{4}$.

Section 1: Activity 4

1. $\frac{7}{12}\div 7=\frac{7}{12}\times\frac{1}{7}$

$$=\frac{\cancel{7}\times 1}{12\times\cancel{7}}$$
$$=\frac{1}{12}$$

Each person will receive $\frac{1}{12}$ of the carton of eggs.

2. $\frac{3}{4}\div 9=\frac{3}{4}\times\frac{1}{9}$

$$=\frac{\overset{1}{\cancel{3}}\times 1}{4\times\underset{3}{\cancel{9}}}$$
$$=\frac{1}{12}$$

Myrtle will spend $\frac{1}{12}$ h on each task.

3. a. The quotient of two fractions is less than 1 when the divisor is greater than the dividend.

b. The quotient of two fractions is greater than 1 when the divisor is less than the dividend.

4. a. $\frac{5}{6}\div\frac{1}{6}=\frac{5}{6}\times\frac{6}{1}$

$$=\frac{5\times\cancel{6}}{\cancel{6}\times 1}$$
$$=\frac{5}{1}$$
$$=5$$

If the servings are each $\frac{1}{6}$ of a cake, 5 servings can be made.

b. $\frac{5}{6}\div\frac{1}{3}=\frac{5}{6}\times\frac{3}{1}$

$$=\frac{5\times\overset{1}{\cancel{3}}}{\underset{2}{\cancel{6}}\times 1}$$
$$=\frac{5}{2}$$
$$=2\frac{1}{2}$$

If the servings are each $\frac{1}{3}$ of a cake, $2\frac{1}{2}$ servings can be made.

c. $\frac{5}{6} \div \frac{1}{4} = \frac{5}{6} \times \frac{4}{1}$

$= \frac{5 \times \cancel{4}^{2}}{\cancel{6}_{3} \times 1}$

$= \frac{10}{3}$

$= 3\frac{1}{3}$

If the servings are each $\frac{1}{4}$ of a cake, $3\frac{1}{3}$ servings can be made.

5. a. $\frac{3}{4} \div \frac{1}{4} = \frac{3}{4} \times \frac{4}{1}$

$= \frac{3 \times \cancel{4}}{\cancel{4} \times 1}$

$= \frac{3}{1}$

$= 3$

If there is $\frac{3}{4}$ of a tank of gas, Roger's father can make 3 trips between the office and the cottage.

b. $\frac{1}{8} \div \frac{1}{4} = \frac{1}{8} \times \frac{4}{1}$

$= \frac{1 \times \cancel{4}^{1}}{\cancel{8}_{2} \times 1}$

$= \frac{1}{2}$

If there is $\frac{1}{8}$ of a tank of gas, Roger's father can make $\frac{1}{2}$ of a trip between the office and the cottage.

c. $\frac{1}{2} \div \frac{1}{4} = \frac{1}{2} \times \frac{4}{1}$

$= \frac{1 \times \cancel{4}^{2}}{\cancel{2}_{1} \times 1}$

$= \frac{2}{1}$

$= 2$

If there is $\frac{1}{2}$ of a tank of gas, Roger's father can make 2 trips between the office and the cottage.

6. a. $\frac{5}{8} \div \frac{1}{8} = 5$

b. $\frac{1}{3} \div \frac{2}{3} = \frac{1}{2}$

c. $\frac{2}{3} \div 2 = \frac{1}{3}$

d. $\frac{4}{5} \div 2 = \frac{2}{5}$

e. $\frac{3}{5} \div \frac{1}{2} = \frac{6}{5}$

$= 1\frac{1}{5}$

f. $\frac{2}{3} \div \frac{1}{6} = 4$

7. **a.** $2\frac{1}{2} \div \frac{5}{8} = \frac{5}{2} \div \frac{5}{8}$

$= \frac{5}{2} \times \frac{8}{5}$

$= \frac{\overset{1}{\cancel{5}} \times \overset{4}{\cancel{8}}}{\underset{1}{\cancel{2}} \times \underset{1}{\cancel{5}}}$

$= \frac{4}{1}$

$= 4$

b. $6\frac{1}{2} \div 1\frac{3}{4} = \frac{13}{2} \div \frac{7}{4}$

$= \frac{13}{2} \times \frac{4}{7}$

$= \frac{13 \times \overset{2}{\cancel{4}}}{\underset{1}{\cancel{2}} \times 7}$

$= \frac{26}{7}$

$= 3\frac{5}{7}$

c. $8\frac{2}{3} \div 1\frac{1}{3} = \frac{26}{3} \div \frac{4}{3}$

$= \frac{26}{3} \times \frac{3}{4}$

$= \frac{\overset{13}{\cancel{26}} \times \overset{1}{\cancel{3}}}{\underset{1}{\cancel{3}} \times \underset{2}{\cancel{4}}}$

$= \frac{13}{2}$

$= 6\frac{1}{2}$

8. $7\frac{1}{2} \div 1\frac{1}{4} = \frac{15}{2} \div \frac{5}{4}$

$= \frac{15}{2} \times \frac{4}{5}$

$= \frac{\overset{3}{\cancel{15}} \times \overset{2}{\cancel{4}}}{\underset{1}{\cancel{2}} \times \underset{1}{\cancel{5}}}$

$= \frac{6}{1}$

$= 6$

Orville can glue down 6 tiles.

9. **a.** $\frac{\frac{2}{7}}{\frac{5}{7}} = \frac{2}{7} \div \frac{5}{7}$

$= \frac{2}{7} \times \frac{7}{5}$

$= \frac{2 \times \cancel{7}}{\cancel{7} \times 5}$

$= \frac{2}{5}$

b. $\frac{\frac{1}{3}}{\frac{3}{4}} = \frac{1}{3} \div \frac{3}{4}$

$= \frac{1}{3} \times \frac{4}{3}$

$= \frac{1 \times 4}{3 \times 3}$

$= \frac{4}{9}$

c. $\frac{\frac{3}{4}}{\frac{1}{2}} = \frac{3}{4} \div \frac{1}{2}$

$= \frac{3}{4} \times \frac{2}{1}$

$= \frac{3 \times \overset{1}{\cancel{2}}}{\underset{2}{\cancel{4}} \times 1}$

$= \frac{3}{2}$

$= 1\frac{1}{2}$

Now Try This

10. Use a table to eliminate clues and solve the problem.

	Starting Time After Signal	Time it Took to Run 1 km	Total Time
Bear	0 min	$2\frac{1}{4}$ min	$2\frac{1}{4}$ min
Giraffe	$\frac{5}{8}$ min	$1\frac{3}{4}$ min	$2\frac{3}{8}$ min
Monkey	1 min	$1\frac{1}{8}$ min	$2\frac{1}{8}$ min
Rabbit	$\frac{2}{3}$ min	$1\frac{13}{24}$ min	$2\frac{5}{24}$ min
Dog	$\frac{1}{4}$ min	$1\frac{1}{2}$ min	$1\frac{3}{4}$ min

a. Use the total time to determine the order in which the animals finished the race.

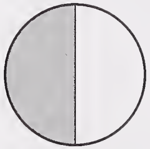
- The dog won the race.
- The monkey came in second.
- The rabbit came in third.
- The giraffe came in last.

b. Use the time it took to run 1 km to determine the speed of the animals.

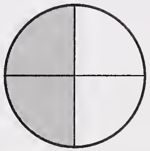
- The monkey ran the fastest.
- The dog ran the second fastest.
- The rabbit ran the third fastest.
- The bear ran the slowest.

Looking Back

11. a. Let the shaded area represent $\frac{1}{2}$.

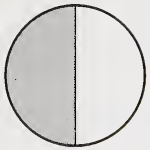


To find the number of $\frac{1}{4}$ pieces in the shaded area, divide the circle into quarters.

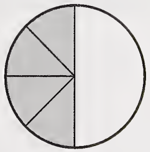


You need to know how many $\frac{1}{4}$ pieces are in the $\frac{1}{2}$. As you can see, there are 2.

b. The father really wanted to say, "Let's divide the remaining $\frac{1}{2}$ pie into fourths." This will give an equal piece for each of the four family members.



When $\frac{1}{2}$ of the pie is divided into 4 pieces, each piece is $\frac{1}{8}$ of the whole pie.



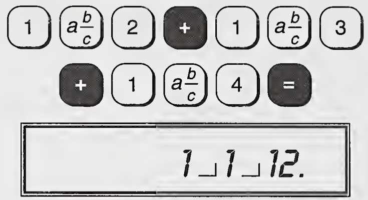
Each person will get a piece equal to $\frac{1}{8}$ of the pie.

Section 1: Activity 5

1. a. **Step 1:** Estimate the answer.

The answer will be between 1 and 2.

Step 2: Press the following key sequence.



Step 3: Compare the calculated answer to the estimate.

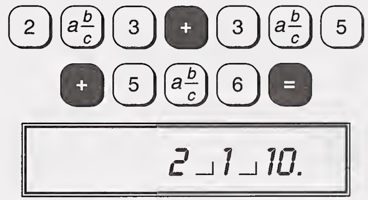
Because $1\frac{1}{12}$ is between 1 and 2, the answer is reasonable.

$$\therefore \frac{1}{2}+\frac{1}{3}+\frac{1}{4}=1\frac{1}{12}$$

b. **Step 1:** Estimate the answer.

The answer will be between 1 and 3.

Step 2: Press the following key sequence.



Step 3: Compare the calculated answer to the estimate.

Because $2\frac{1}{10}$ is between 1 and 3, the answer is reasonable.

$$\therefore \ \frac{2}{3}+\frac{3}{5}+\frac{5}{6}=2\frac{1}{10}$$

2. **a.** Estimation suggests the answer will be approximately 9.

6 $a\frac{b}{c}$ 9 $a\frac{b}{c}$ 1 3 +
2 $a\frac{b}{c}$ 1 $a\frac{b}{c}$ 5 =

8 ⌟ 58 ⌟ 65.

The result, $8\frac{58}{65}$, is close to the estimate.

$$\therefore \ 6\frac{9}{13}+2\frac{1}{5}=8\frac{58}{65}$$

b. Estimation suggests the answer will approximate 157.

1 4 8 $a\frac{b}{c}$ 1 $a\frac{b}{c}$ 9
+ 8 $a\frac{b}{c}$ 5 $a\frac{b}{c}$ 6 =

156 ⌟ 17 ⌟ 18.

The result, $156\frac{17}{18}$, is close to the estimate.

$$\therefore \ 148\frac{1}{9}+8\frac{5}{6}=156\frac{17}{18}$$

3. **a.** Estimation suggests the answer will be close to 1.

3 $a\frac{b}{c}$ 1 $a\frac{b}{c}$ 3 − 2 $a\frac{b}{c}$ 1 $a\frac{b}{c}$ 2 =

5 ⌟ 6.

The result, $\frac{5}{6}$, is close to the estimate.

$$\therefore \ 3\frac{1}{3}-2\frac{1}{2}=\frac{5}{6}$$

Dave ran $\frac{5}{6}$ km further on Wednesday.

b. Estimation suggests Azra will have approximately 3 cups of flour left.

7 $a\frac{b}{c}$ 1 $a\frac{b}{c}$ 2 − 4 $a\frac{b}{c}$ 3 $a\frac{b}{c}$ 4 =

2 ⌟ 3 ⌟ 4.

The result, $2\frac{3}{4}$, is close to the estimate.

$\therefore\ 7\frac{1}{2} - 4\frac{3}{4} = 2\frac{3}{4}$

Since Azra has $2\frac{3}{4}$ cups of flour left over, she will have enough to make the cake that calls for $2\frac{1}{2}$ cups.

4. a. Estimation suggests the answer is approximately 9.

7 $a\frac{b}{c}$ 7 $a\frac{b}{c}$ 9 + 1 $a\frac{b}{c}$ 1 $a\frac{b}{c}$ 3 =

9 ⌟ 1 ⌟ 9.

$$\therefore\ \begin{array}{r} 7\frac{7}{9} \\ +\ 1\frac{1}{3} \\ \hline 9\frac{1}{9} \end{array}$$

b. Estimation suggests the answer is approximately 8 or 9.

2 $a\frac{b}{c}$ 7 $a\frac{b}{c}$ 8 + 5 $a\frac{b}{c}$ 1 $a\frac{b}{c}$ 2 =

8 ⌟ 3 ⌟ 8.

$\therefore\ 2\frac{7}{8} + 5\frac{1}{2} = 8\frac{3}{8}$

c. Estimation suggests the answer is approximately 20.

2 7 $a\frac{b}{c}$ 2 $a\frac{b}{c}$ 3 − 6 $a\frac{b}{c}$ 7 $a\frac{b}{c}$ 9 =

20 ⌟ 8 ⌟ 9.

$$\therefore\ \begin{array}{r} 27\frac{2}{3} \\ -\ 6\frac{7}{8} \\ \hline 20\frac{8}{9} \end{array}$$

d. Estimate.

The fraction $\frac{39}{85}$ is close to $\frac{40}{80}$ or $\frac{1}{2}$. The fraction $\frac{2}{15}$ is close to $\frac{2}{16}$, which simplifies to $\frac{1}{8}$.

$$\therefore \frac{1}{2}-\frac{1}{8}=\frac{4}{8}-\frac{1}{8}$$
$$=\frac{3}{8}$$

The answer should be close to $\frac{3}{8}$.

3 9 $a\frac{b}{c}$ 8 5 − 2 $a\frac{b}{c}$ 1 5 =

83 ⌟ 255.

$$\therefore \frac{39}{85}-\frac{2}{15}=\frac{83}{255}$$

e. Estimate.

The answer will be more than 6×2 or 12. Rounding each to the next integer gives 7×3 or 21. Therefore, the answer will be between 12 and 21.

6 $a\frac{b}{c}$ 3 $a\frac{b}{c}$ 5 × 2 $a\frac{b}{c}$ 7 $a\frac{b}{c}$ 9 =

18 ⌟ 1 ⌟ 3.

$$\therefore 6\frac{3}{5}\times 2\frac{7}{9}=18\frac{1}{3}$$

f. Estimate.

The fraction $\frac{27}{10}$ is close to 3. Multiply $5\frac{1}{2}$ by 3.

$$5\times 3=15 \qquad \frac{1}{2}\times 3=1\frac{1}{2}$$

$$\therefore 15+1\frac{1}{2}=16\frac{1}{2}$$

The answer will be less than $16\frac{1}{2}$.

2 7 $a\frac{b}{c}$ 1 0 × 5 $a\frac{b}{c}$ 1 $a\frac{b}{c}$ 2 =

14 ⌟ 17 ⌟ 20.

$$\therefore \frac{27}{10}\times 5\frac{1}{2}=14\frac{17}{20}$$

g. Estimate.

The fraction $\frac{12}{25}$ is close to $\frac{12}{24}$ or $\frac{1}{2}$. The fraction $\frac{4}{15}$ is close to $\frac{4}{16}$ or $\frac{1}{4}$.

$$\therefore \frac{1}{2} \div \frac{1}{4} = \frac{1}{2} \times \frac{4}{1}$$
$$= 2$$

The answer will be approximately 2.

`1` `2` `a b/c` `2` `5` `÷` `4` `a b/c` `1` `5` `=`

`1⌟4⌟5.`

$$\therefore \frac{12}{25} \div \frac{4}{15} = 1\frac{4}{5}$$

h. Estimate.

The answer will be somewhere around $8 \div 2 = 4$.

`8` `a b/c` `2` `a b/c` `3` `÷` `2` `a b/c` `3` `a b/c` `5` `=`

`3⌟1⌟3.`

$$\therefore 8\frac{2}{3} \div 2\frac{3}{5} = 3\frac{1}{3}$$

5. a. Estimate.

$5 \times 7 = 35$

Since the fractions make each factor larger, the answer will be more than 35.

`5` `a b/c` `1` `a b/c` `4` `×` `7` `a b/c` `1` `a b/c` `4` `=`

`38⌟1⌟16.`

The first square has $38\frac{1}{16}$ stitches.

b. Estimate.

$6 \times 5 = 30$

Since these numbers were rounded up, the product will be less than 30.

`5` `a b/c` `1` `a b/c` `2` `×` `4` `a b/c` `1` `a b/c` `2` `=`

`24⌟3⌟4.`

The second square has $24\frac{3}{4}$ stitches.

Looking Back

6. a. 6 × 1 $a\frac{b}{c}$ 4 =

1⌟1⌟2.

Andrew will need $1\frac{1}{2}$ kg of hamburger.

b. 3 $a\frac{b}{c}$ 4 + 7 $a\frac{b}{c}$ 8 =

1⌟5⌟8.

The recipe requires $1\frac{1}{2}$ kg. With the two packages, Andrew has $1\frac{5}{8}$ kg. Therefore, he does have enough hamburger.

7. a. Mental math and paper and pencil works best on computations involving common fractions (e.g., $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$).

b. A calculator might be more useful with uncommon fractions and those with larger numbers for numerators and denominators (e.g., $\frac{8}{9}$, $\frac{29}{57}$, and $\frac{17}{83}$). It might also be easier to use a calculator for division, since the reciprocal is not needed.

Section 2: Activity 1

1. a. The ratio of Franz's height to his father's height is 199 to 175.

b. The ratio of Franz's height to his mother's height is 199 to 167.

2. a. The ratio of Emma's age to Mavis's age is 14 to 15.

b. The ratio of Mavis's age to Emma's age is 15 to 14.

3. a. The win-loss ratio is 41 to 10.
The win-loss ratio is $41:10$.

The win-loss ratio is $\frac{41}{10}$.

b. The ratio of the number of wins to the total number of games played is 41 to 59.

The ratio of the number of wins to the total number of games played is $41:59$.

The ratio of the number of wins to the total number of games played is $\frac{41}{59}$.

4. $\frac{200}{100} \overset{\div 100}{=} \frac{2}{1}$ $\frac{\text{amount in larger tube (mL)}}{\text{amount in smaller tube (mL)}}$

The ratio of the amount of toothpaste in the larger tube to the amount of toothpaste in the smaller tube is $2:1$.

5. a. $\overset{\div 150}{\dfrac{300}{450}} = \underset{\div 150}{\dfrac{2}{3}}$ $\dfrac{\text{mass of gravel (kg)}}{\text{mass of sand (kg)}}$

The ratio of the mass of gravel to the mass of sand is $2:3$.

b. $\overset{\div 100}{\dfrac{300}{100}} = \underset{\div 100}{\dfrac{3}{1}}$ $\dfrac{\text{mass of gravel (kg)}}{\text{mass of cement (kg)}}$

The ratio of the mass of gravel to the mass of cement is $3:1$.

6. a. **Step 1:** Find the total number of medals won by West Germany in the 1988 Winter Olympics.

$$2+4+2=8$$

Step 2: State the ratio of gold medals to total medals.

$$2:8$$

Step 3: Write the ratio in lowest terms.

$$\overset{\div 2}{\frac{2}{8}} = \underset{\div 2}{\frac{1}{4}}$$

The ratio of the number of gold medals won by West Germany to the total number of medals won by West Germany in the 1988 Winter Olympics is $1:4$.

b. **Step 1:** Find the total number of medals won by Switzerland in the 1988 Winter Olympics.

$$5+5+5=15$$

Step 2: State the ratio of gold medals to total medals.

$$5:15$$

Step 3: Write the ratio in lowest terms.

$$\overset{\div 5}{\frac{5}{15}} = \underset{\div 5}{\frac{1}{3}}$$

The ratio of the number of gold medals won by Switzerland to the total number of medals won by Switzerland in the 1988 Winter Olympics is $1:3$.

7. **Step 1:** Write the measurements in same unit. Convert the mass of the pumpkin to grams.

$$2 \text{ kg} = 2000 \text{ g}$$

Step 2: Write the ratio in lowest terms.

$\overset{\div 250}{\dfrac{2000}{250}} = \underset{\div 250}{\dfrac{8}{1}}$ $\dfrac{\text{mass of pumpkin (g)}}{\text{mass of zucchini (g)}}$

The ratio of the mass of the pumpkin to the mass of the zucchini is $8:1$.

8. **Step 1:** Write the measurements in the same unit. Convert the amount of blood in the body to millilitres.

$$5 \text{ L} = 5000 \text{ mL}$$

Step 2: Write the ratio in lowest terms.

$$\frac{450}{5000} \overset{\div 50}{=} \frac{9}{100} \qquad \frac{\text{amount of blood donated (mL)}}{\text{amount of blood in the body (mL)}}$$

The ratio of the amount of blood donated to the amount of blood in the entire body is $9:100$.

9. The ratio of the amount of concentrate to the amount of water is $\frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \frac{4}{12}, \ldots$.

10. The ratio of the recommended number of students to the number of teachers is $\frac{20}{1}, \frac{40}{2}, \frac{60}{3}, \frac{80}{4}, \ldots$.

Now Try This

11. You may find it helpful to use diagrams or objects to solve this problem.

   a. Fill the 5-L pail with water. Gradually pour the water from the 5-L pail into the 3-L pail until the 3-L pail is full.

   There is exactly 2 L left in the 5-L pail.

   b. **Step 1:** Fill the 3-L pail with water. Pour the water from the 3-L pail into the 5-L pail.

   Step 2: Fill the 3-L pail with water again. Gradually pour the water from the 3-L pail into the 5-L pail until the 5-L pail is full.

   There is exactly 1 L left in the 3-L pail.

   c. **Step 1:** Fill the 3-L pail with water. Pour the water from the 3-L pail into the 5-L pail.

   Step 2: Fill the 3-L pail with water. Gradually pour the water from the 3-L pail into the 5-L pail until the 5-L pail is full. There will be 1 L left in the 3-L pail.

   Step 3: Empty the water from the 5-L pail. Pour the water from the 3-L pail into the 5-L pail. There is now 1 L of water in the 5-L pail.

   Step 4: Fill the 3-L pail with water. Pour the water from the 3-L pail into the 5-L pail.

   There are now exactly 4 L of water in the 5-L pail.

Looking Back

12. The directions mean the ratio of water to rice is $2:1$. For example, if you use 4 cups of water, you would need 2 cups of rice. Or, if you use $\frac{1}{2}$ cup of rice, you would need 1 cup of water. The amount of water needed is always double the amount of rice.

Section 2: Activity 2

1. a. The rate of cost is \$3.99 per 12 doughnuts.

   b. The rate of cost is \$3.99 : 12 doughnuts.

   c. The rate of cost is $\frac{\$3.99}{12 \text{ doughnuts}}$.

   d. The rate of cost is \$3.99/12 doughnuts.

2. a. The rate of stretch is 1 cm per 25 g.

   b. The rate of stretch is 1 cm : 25 g.

   c. The rate of stretch is $\frac{1 \text{ cm}}{25 \text{ g}}$.

   d. The rate of stretch is 1 cm/25 g.

3. a. The rate of coverage is 4 L per 25 m^2.

   b. The rate of coverage is 4 L : 25 m^2.

   c. The rate of coverage is $\frac{4 \text{ L}}{25 \text{ m}^2}$.

   d. The rate of coverage is 4 L/25 m^2.

4. a. $\underset{\div 4}{\overset{\div 4}{\frac{420}{4} = \frac{105}{1}}}$ $\quad \frac{\text{number of words}}{\text{amount of time (min)}}$

   Bobby's typing rate is 105 words per minute.

   b. $\underset{\div 4}{\overset{\div 4}{\frac{300}{4} = \frac{75}{1}}}$ $\quad \frac{\text{number of beats}}{\text{amount of time (min)}}$

   Dorothy's heart rate is 75 beats per minute.

   c. $\underset{\div 100}{\overset{\div 100}{\frac{2}{100} = \frac{0.02}{1}}}$ $\quad \frac{\text{amount of stretch (cm)}}{\text{amount of mass (g)}}$

   The rate of stretch is 0.02 cm per gram.

   d. $\underset{\div 100}{\overset{\div 100}{\frac{215}{100} = \frac{2.15}{1}}}$ $\quad \frac{\text{amount in Canadian dollars}}{\text{amount in British pounds}}$

   The exchange rate is about CDN\$2.15 per British pound.

5. Arlene's rate of speed is 2.5 m/s, 5 m/2 s, 7.5 m/3 s, 10 m/4 s, … .

6. $\frac{71\,528}{29\,672} \doteq \frac{2.41}{1000}$ (÷ 29 672)

$\frac{\text{number of divorces}}{\text{total number of people}}$

The divorce rate for Canada in 1996 was approximately 2.41 per 1000 population.

7. **Photo A** **Photo B**

$\frac{40}{32} = \frac{5}{4}$ (÷ 8) $\frac{50}{40} = \frac{5}{4}$ (÷ 10)

$\frac{\text{length (mm)}}{\text{width (mm)}}$

The ratio of the lengths to the widths of the photographs are equivalent.

8. **Cylinder A** **Cylinder B**

$\frac{2}{3}$ $\frac{3}{2}$

$\frac{\text{height (cm)}}{\text{diameter (cm)}}$

Because $\frac{2}{3} \neq \frac{3}{2}$, the ratios of the heights to the diameters of the cylinders are not equivalent.

9. **Dino** **Frank**

$\frac{830}{9} \doteq \frac{92.2}{1}$ $\frac{640}{7} \doteq \frac{91.4}{1}$

$\frac{\text{distance (km)}}{\text{time (h)}}$

Because $\frac{92.2}{1} > \frac{91.4}{1}$, Dino travelled at a faster rate of speed.

10. **2 Servings** **4 Servings** **6 Servings**

$\frac{125}{300} = \frac{5}{12}$ $\frac{175}{375} = \frac{7}{15}$ $\frac{250}{500} = \frac{1}{2}$

$\doteq 0.41$ $\doteq 0.47$ $= 0.5$

$\frac{\text{rice (mL)}}{\text{water (mL)}}$

The ratio of the amount of rice to the amount of water is not proportional for 2, 4, and 6 servings.

11. a. **Dressing** **Situation A**

$\frac{2}{1}$ $\frac{60}{30} = \frac{2}{1}$

$\frac{\text{oil (mL)}}{\text{vinegar (mL)}}$

Yes, this could be the amount of oil and vinegar in the salad dressing because the ratios are proportional.

b. **Dressing** **Situation B**

$\frac{2}{1}$ $\frac{30}{60} = \frac{1}{2}$

$\frac{\text{oil (mL)}}{\text{vinegar (mL)}}$

No, this could not be the amount of oil and vinegar in the salad dressing because the ratios are not proportional.

c. **Dressing** — **Situation C**

$\frac{2}{1}$ $\qquad$ $\frac{90}{45}=\frac{2}{1}$ $\qquad$ $\frac{\text{oil (mL)}}{\text{vinegar (mL)}}$

Yes, this could be the amount of oil and vinegar in the salad dressing because the ratios are proportional.

d. **Dressing** — **Situation D**

$\frac{2}{1}$ $\qquad$ $\frac{45}{90}=\frac{1}{2}$ $\qquad$ $\frac{\text{oil (mL)}}{\text{vinegar (mL)}}$

No, this could not be the amount of oil and vinegar in the salad dressing because the ratios are not proportional.

12. **Rate 1** — **Rate 2**

$\frac{8.60}{2}=\frac{4.30}{1}$ $\qquad$ $\frac{17.20}{4}=\frac{4.30}{1}$ $\qquad$ $\frac{\text{cost (\$)}}{\text{mass (kg)}}$

Rate 3

$\frac{25.80}{6}=\frac{4.30}{1}$

Yes, the rates are proportional.

Looking Back

13. There are many rates used and seen daily. Here are some examples:

- km/h (speed)
- \$/h (hourly wage)
- \$/12 (the cost per dozen)
- m/s (metres per second—speed of running or swimming)

14. The reason 1000 was chosen is because people can imagine 5 marriages per 1000 or 2 divorces per 1000. Most people would have a difficult time visualizing 0.002 divorces per person. It is easier to understand if the number is between 0 and 100.

15. $\frac{430\ 000}{21} \doteq \frac{20\ 476}{1}$ $\qquad$ $\frac{\text{population}}{\text{area}\left(\text{km}^2\right)}$

Macau's population is 20 476 people/ km^2.

Section 2: Activity 3

1. a. $\frac{20}{100}=\frac{1}{5}$

The number of raisins is $\frac{1}{5}$ of the number of bran flakes.

b. $\frac{20}{100} = 0.20$

$= 0.2$

The number of raisins is 0.2 of the number of bran flakes.

c. $\frac{20}{100} = 20\%$

The number of raisins is 20% of the number of bran flakes.

2. a. $\frac{5}{100} = \frac{1}{20}$

The length of a grasshopper is $\frac{1}{20}$ of the distance the grasshopper can jump.

b. $\frac{5}{100} = 0.05$

The length of a grasshopper's body is 0.05 of the distance the grasshopper can jump.

c. $\frac{5}{100} = 5\%$

The length of a grasshopper's body is 5% of the distance the grasshopper can jump.

3. $100\% - 30\% = 70\%$

Ramon lost 70% of the ski races he entered.

4. $100\% - 5\% = 95\%$

In the group, 95% of the children are right-handed.

5. $100\% - 20\% = 80\%$

In the group, 80% do not wear glasses.

6. a. The number of students present is $\frac{3}{4}$ of the number of students in the class.

b. $\frac{3}{4} = \frac{75}{100}$

$= 0.75$

The number of students present is 0.75 of the number of students in the class.

c. $\frac{3}{4} = \frac{75}{100}$

$= 75\%$

The number of students present is 75% of the number of students in the class.

7. a. The length of the nail is $\frac{3}{5}$ of the length of the pencil.

b. $\frac{3}{5} = \frac{6}{10}$
$= 0.6$

The length of the nail is 0.6 of the length of the pencil.

c. $\frac{3}{5} = \frac{60}{100}$
$= 60\%$

The length of the nail is 60% of the length of the pencil.

8. $\overset{\times 12.5}{\underset{\times 12.5}{\frac{5}{8} = \frac{62.5}{100}}}$

In Vasili's neighbourhood, 62.5% of the pets are dogs.

9. **a.** $\frac{375}{375} = \frac{1}{1}$ $\quad \frac{\text{amount of cranberry juice (mL)}}{\text{amount of apple juice (mL)}}$

The ratio of the amount of cranberry juice to the amount of apple juice is 1 to 1.

b. $\frac{375}{375} = \frac{1}{1}$ $\quad \frac{\text{amount of cranberry juice (mL)}}{\text{amount of apple juice (mL)}}$
$= 1$

The amount of cranberry juice is 1 times the amount of apple juice.

c. $\frac{375}{375} = \frac{1}{1}$ $\quad \frac{\text{amount of cranberry juice (mL)}}{\text{amount of apple juice (mL)}}$
$= \frac{100}{100}$
$= 100\%$

The amount of cranberry juice is 100% of the amount of apple juice.

10. **a.** $\frac{12}{2} = \frac{6}{1}$ $\quad \frac{\text{height of jump on the Moon (m)}}{\text{height of jump on Earth (m)}}$

The ratio of the height of the jump on the Moon to the height of the jump on Earth is $6:1$.

b. $\frac{12}{2} = \frac{6}{1}$ $\quad \frac{\text{height of jump on the Moon (m)}}{\text{height of jump on Earth (m)}}$
$= 6 \div 1$
$= 6$

The height of the jump on the Moon is 6 times the height of the jump on Earth.

c. $\frac{12}{2} = \frac{6}{1}$

$= \frac{600}{100}$

$= 600\%$

$\frac{\text{height of jump on the Moon (m)}}{\text{height of jump on Earth (m)}}$

The height of the jump on the Moon is 600% of the height of the jump on Earth.

11. $6 = \frac{6}{1}$

$= \frac{600}{100}$

$= 600\%$

Some jet planes travel at a speed that is 600% of the flying speed of the spine-tail swift.

12. $2 = \frac{2}{1}$

$= \frac{200}{100}$

$= 200\%$

A supersonic jet can fly at a speed that is 200% of the speed of sound.

13. $2700 = \frac{2700}{1}$

$= \frac{270\,000}{100}$

$= 270\,000\%$

The mass of an ostrich egg is 270 000% of the mass of a hummingbird egg.

14. a. $\frac{24}{15} = \frac{8}{5}$

$\frac{\text{number of canoes}}{\text{number of sailboats}}$

The ratio of the number of canoes on the lake to the number of sailboats is 8 to 5.

b. $\frac{24}{15} = \frac{8}{5}$

$= 1\frac{3}{5}$

$\frac{\text{number of canoes}}{\text{number of sailboats}}$

The number of canoes on the lake is $1\frac{3}{5}$ times the number of sailboats.

c. $\frac{24}{15} = \frac{8}{5}$

$= 1.6$

$\frac{\text{number of canoes}}{\text{number of sailboats}}$

The number of canoes on the lake is 1.6 times the number of sailboats.

d. $\frac{24}{15} = \frac{8}{5}$

$= \frac{160}{100}$

$= 160\%$

$\frac{\text{number of canoes}}{\text{number of sailboats}}$

The ratio of the number of canoes on the lake is 160% of the number of sailboats.

15. $0.75 = \frac{75}{100}$

$= 75\%$

The price of advanced tickets is 75% of the price of tickets at the door.

16. $0.390 = 0.39$

$= \frac{39}{100}$

$= 39\%$

In 1914, Ty Cobb hit safely 39% of the time he was up to bat.

17. $1.56 = \frac{1.56}{1}$

$= \frac{156}{100}$

$= 156\%$

The mass of milk is 156% of the mass of gasoline.

18. **a.** $20\% = \frac{20}{100}$

$= 0.20$

$= 0.2$

The students kept 0.2 of the sales.

b. $20\% = \frac{20}{100}$

$= \frac{1}{5}$

The students kept $\frac{1}{5}$ of the sales.

19. **a.** $35\% = \frac{35}{100}$

$= 0.35$

Darryl made 0.35 of the shots he attempted.

b. $35\% = \frac{35}{100}$

$= \frac{7}{20}$

Darryl made $\frac{7}{20}$ of the shots he attempted.

20. **Test 1**

$\frac{17}{25} = \frac{68}{100}$

$= 68\%$

Test 2

$\frac{14}{20} = \frac{70}{100}$

$= 70\%$

$\frac{\text{score}}{\text{total}}$

Kara did better on the second test.

21. **Haden**

$\frac{7}{10} = \frac{70}{100}$

$= 70\%$

Steven

$\frac{13}{20} = \frac{65}{100}$

$= 65\%$

$\frac{\text{shots}}{\text{attempts}}$

Haden is the more accurate shooter.

22. **a.** $\frac{6}{12} = \frac{1}{2}$

$\frac{\text{number of teeth on driving gea}}{\text{number of teeth on driven wheel}}$

The gear ratio is 1 : 2.

b. Gear A makes 2 complete revolutions.
Gear B makes 1 complete revolution.

c. $\frac{2}{1}$

$\frac{\text{number of turns on driving gea}}{\text{number of turns on driven gear}}$

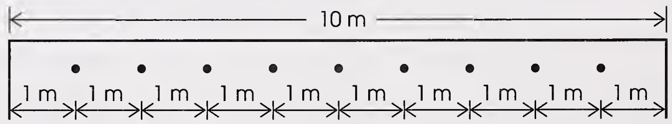
The turn ratio is 2 : 1.

23.

Gear	Number of Teeth on Front Gear	Number of Teeth on Back Gear	Gear Ratio	Turn Ratio
1st	52	28	1.857	0.538
2nd	52	24	2.167	0.462
3rd	52	20	2.600	0.385
4th	52	17	3.059	0.327
5th	52	14	3.714	0.269

Now Try This

24. To solve this problem, you may use objects or make a diagram.



If the hooks are placed every metre, 9 hooks are needed.

Looking Back

25.

Fraction	Decimal	Percent
$\frac{9}{20}$	0.45	45%
$\frac{15}{100} = \frac{3}{20}$	0.15	15%
$\frac{55}{100} = \frac{11}{20}$	0.55	55%

To convert any fraction to a decimal, divide the numerator by the denominator using long division or a calculator.

To convert a fraction to a percent, find an equivalent ratio with a denominator of 100 or convert the fraction to a decimal and rewrite as a percent.

To convert a decimal to a fraction, look at place value (tenths, hundredths, …) and use that as the denominator. The fraction may need to be simplified to lowest terms.

$$0.167 = \frac{167}{1000} \text{ and } 0.52 = \frac{52}{100}$$

To convert a decimal to a percent, write the numbers in the tenths and hundredths place. Follow them with a decimal. Numbers in the thousandths, ten thousandths, and so on follow.

$0.5625 = 56.25\%$ and $1.521 = 152.1\%$

To convert a percent to a decimal, divide the percent by 100. To convert a percent to a fraction, omit the percent sign and write that number as the numerator over the denominator of 100. Simplify if necessary.

Section 2: Activity 4

1. $\frac{2}{1} = \frac{350}{\square}$ $\qquad \frac{2 \times 175}{1 \times 175} = \frac{350}{\boxed{175}}$ $\qquad \dfrac{\text{amount of flour (mL)}}{\text{amount of sugar (mL)}}$

Tom needs 175 mL of sugar.

2. $\frac{3}{5} = \frac{9}{\square}$ $\qquad \frac{3 \times 3}{5 \times 3} = \frac{9}{\boxed{15}}$ $\qquad \dfrac{\text{number of cooks}}{\text{number of waiters and waitresses}}$

There are 15 waiters and waitresses.

3. $\frac{7}{2} = \frac{21}{\square}$ $\qquad \frac{6 \times 3}{12 \times 3} = \frac{24}{\boxed{48}}$ $\qquad \dfrac{\text{number of canoes}}{\text{number of sailboats}}$

There are 6 sailboats.

4. $\frac{1}{7} \doteq \frac{\square}{206}$ $\qquad \frac{1 \times 29}{7 \times 29} \doteq \frac{\boxed{29}}{206}$ $\qquad \dfrac{\text{number of bones in the head}}{\text{total number of bones}}$

There are about 29 bones in the head.

5. $\frac{37}{40} = \frac{\square}{400}$ $\qquad \frac{37}{40} \overset{\times 10}{\underset{\times 10}{=}} \frac{370}{400}$ $\qquad \frac{\text{mass of silver (g)}}{\text{total mass (g)}}$

There are 370 g of silver in the cup.

6. $\frac{2.5}{1} = \frac{\square}{50}$ $\qquad \frac{2.5}{1} \overset{\times 50}{\underset{\times 50}{=}} \frac{125}{50}$ $\qquad \frac{\text{distance (m)}}{\text{time (s)}}$

At this rate, Arlene can swim 125 m.

7. **Step 1:** Simplify the rate.

$$\frac{450}{40} = \frac{45}{4}$$

Step 2: Write a proportion and find the missing term.

$$\frac{45}{4} \overset{\times 9}{\underset{\times 9}{=}} \frac{405}{36} \qquad \frac{\text{earnings (\$)}}{\text{time (h)}}$$

At this rate, René will earn $405.

8. **Step 1:** Simplify the rate.

$$\frac{25}{2.5} = \frac{10}{1}$$

Step 2: To solve the problem, write a proportion and find the missing term.

$$\frac{10}{1} \overset{\times 5}{\underset{\times 5}{=}} \frac{50}{5} \qquad \frac{\text{distance (km)}}{\text{time (h)}}$$

At this rate, Chester will need 5 h to travel 50 km.

9. **Step 1:** Simplify the rate.

$$\frac{1000}{14} = \frac{500}{7}$$

Step 2: To solve the problem, write a proportion and find the missing term.

$$\frac{500}{7} \overset{\times 10}{\underset{\times 10}{=}} \frac{5000}{70} \qquad \frac{\text{amount (Japanese yen)}}{\text{amount (Canadian dollars)}}$$

You can buy 5000 Japanese yen for $70 Canadian.

10. **Step 1:** Write the percent as a ratio in simplest form.

$$23\% = \frac{23}{100}$$

Step 2: Write a proportion and find the missing term.

$$\frac{23}{100} \overset{\times 2}{=} \frac{\boxed{46}}{200}$$

$$\frac{\text{amount of milk fat (g)}}{\text{total mass (g)}}$$

There is 46 g of milk fat in the package of cheese.

11. **Step 1:** Write the percent as a ratio in simplest form.

$$15\% = \frac{15}{100} = \frac{3}{20}$$

Step 2: Write a proportion and find the missing term.

$$\frac{3}{20} \overset{\times 3}{=} \frac{9}{\boxed{60}}$$

$$\frac{\text{amount of tip (\$)}}{\text{amount of bill (\$)}}$$

The Allans's food bill was $60.

12. **Step 1:** Write the percent as a ratio in simplest form.

$$130\% = \frac{130}{100} = \frac{13}{10}$$

Step 2: Write a proportion and find the missing term.

$$\frac{13}{10} \overset{\times 520}{=} \frac{\boxed{6760}}{5200}$$

$$\frac{\text{profits this year (\$)}}{\text{profits last year (\$)}}$$

The profits this year were $6760.

13. **Step 1:** Write the percent as a ratio in simplest form.

$$80\% = \frac{80}{100} = \frac{4}{5}$$

Step 2: Write a proportion and find the missing term.

$$\frac{4}{5} \overset{\times 30}{=} \frac{120}{\boxed{150}}$$

$$\frac{\text{number of students with part-time jobs}}{\text{total number of students}}$$

There are 150 students in the school.

Now Try This

14. Step 1: Find the percent of cattle sold to the fourth buyer.

$$50\% + 25\% + 20\% = 95\%$$

So, 95% of the cattle were sold to the first three buyers.

Therefore, $100\% - 95\% = 5\%$ of the cattle were sold to the fourth buyer.

Step 2: Write the percent as a ratio in simplest form.

$$\begin{aligned} 5\% &= \frac{5}{100} \\ &= \frac{1}{20} \end{aligned}$$

Step 3: To solve the problem, write a proportion and find the missing term.

$$\frac{1}{20} \underset{\times 7}{\overset{\times 7}{=}} \frac{7}{\boxed{140}} \qquad \frac{\text{number of cattle sold to the fourth buyer}}{\text{total number of cattle}}$$

The farmer had 140 cattle.

15. Step 1: Write the percent as a ratio in simplest form.

$$\begin{aligned} 80\% &= \frac{80}{100} \\ &= \frac{4}{5} \end{aligned}$$

Step 2: Find the ratio of the correct responses to the number of incorrect responses. Use this reasoning.

If the ratio of the number of correct responses to the total number of responses is **4** to 5, the ratio of the number of incorrect responses to the total number of responses is **1** to 5.

Therefore, the ratio of the number of correct responses to the number of incorrect responses is 4 to 1.

16. Step 1: Find Geoff's race time. **Hint:** If Geoff gets a 10-m start in the 100-m race, he has only 90 m left to run.

$$\frac{5}{1} \underset{\times 18}{\overset{\times 18}{=}} \frac{90}{\boxed{18}} \qquad \frac{\text{distance (m)}}{\text{time (s)}}$$

With a 10-m head start, Geoff would run the 100-m race in 18 s.

Step 2: Find Simon's race time.

$$\frac{7.5}{1} \underset{\times 13.3}{\overset{\times 13.3}{\doteq}} \frac{100}{\boxed{13.3}} \qquad \frac{\text{distance (m)}}{\text{time (s)}}$$

Simon would run the race in about 13.3 s.

Step 3: Compare the race times.

$13.3 < 18$

Geoff would not win a 100-m race with a 10-m head start.

17. **Step 1:** Change the 200-m time to seconds.

$1 \text{ min } 52.8 \text{ s} = 112.8 \text{ s}$

Suki swims the 200-m race in 112.8 s.

Step 2: Find the time for a 200-m race at the 100-m pace. Write a proportion and find the missing term.

$$\frac{100}{51.2} \overset{\times 2}{=} \frac{200}{102.4}$$

$$\frac{\text{distance (m)}}{\text{time (s)}}$$

At this rate, Suki would swim the 200-m race in 102.4 s.

Therefore, Suki would cut 10.4 s from her record.

18. If the number has no remainder when divided by 7, it is a multiple of 7.

The multiples of 7 are $7, 14, 21, 28, 35, 42, 49, 56, \ldots$.

Use the guess, check, and revise strategy to find the number.

Guess 1: The number is 7.

$7 \div 2 = 3 \text{ R1}$

$7 \div 3 = 2 \text{ R1}$

$7 \div 4 = 1 \text{ R3}$ ← This does not meet the condition.

Guess 2: The number is 14.

$14 \div 2 = 7$ ← This does not meet the condition.

Guess 3: The number is 21.

$21 \div 2 = 10 \text{ R1}$

$21 \div 3 = 7$ ← This does not meet the condition.

Guess 4: The number is 28.

$28 \div 2 = 14$ ← This does not meet the condition.

Guess 5: The number is 35.

$35 \div 2 = 17 \text{ R1}$

$35 \div 3 = 11 \text{ R2}$ ← This does not meet the condition.

Guess 6: The number is 42.

$42 \div 2 = 21$ ← This does not meet the condition.

Guess 7: The number is 49.

$49 \div 2 = 24 \text{ R}1$

$49 \div 3 = 16 \text{ R}1$

$49 \div 4 = 22 \text{ R}1$

$49 \div 6 = 8 \text{ R}1$

$49 \div 7 = 7 \text{ R}0$ ← This meets the condition.

All conditions are met. So, the number is 49.

Looking Back

19. First, calculate the cost of the bike this week.

$$\begin{aligned}\text{Cost} &= 720 - 140\\ &= 580\end{aligned}$$

This week, the bike will cost \$580.

Now, calculate the possible costs of the bike next week.

Scratch and Save 10%

$$\overset{\times 7.2}{\frac{10}{100} = \frac{72}{720}}$$

$$\begin{aligned}\therefore\ \text{Cost} &= 720 - 72\\ &= \$648\end{aligned}$$

Scratch and Save 20%

$$\overset{\times 7.2}{\frac{20}{100} = \frac{144}{720}}$$

$$\begin{aligned}\therefore\ \text{Cost} &= 720 - 144\\ &= \$576\end{aligned}$$

Scratch and Save 30%

$$\overset{\times 7.2}{\frac{30}{100} = \frac{216}{720}}$$

$$\begin{aligned}\therefore\ \text{Cost} &= 720 - 216\\ &= \$504\end{aligned}$$

Explanations may vary. The sample explanations are given.

If Larisa waits until the "scratch and save" promotion, she may get the 10% discount and would lose $\$648 - \$580 = \$68$ by not buying it now.

If Larisa gets the 20% discount, she would save $580 - 576 = \$4$. If Larisa got the 30% discount, she would save $580 - 504 = \$76$ by waiting. Larisa should buy now because she won't gain much with the 20% discount and she will lose with the 10% discount. There is always a higher chance of getting a 10% discount or 20% discount compared to a 30% discount.

Section 3: Activity 1

1. a. The graph compares the number of passengers at Canada's five busiest airports in 1985.

b. The data is arranged in order of rank from greatest to least.

c.  ($\frac{3}{4}$ of the whole symbol) represents 75 000 passengers;  ($\frac{1}{2}$ of the whole symbol) represents 50 000 passengers; and  ($\frac{1}{4}$ of the whole symbol) represents 25 000 passengers.

d. About 1 500 000 passengers used Toronto's airport.

e. About 725 000 passengers used Vancouver's airport.

f. About 1 300 000 more passengers used Toronto's airport than Winnipeg's airport.

2. a. The percentages of illiterate adults in different areas of the developing world in 1990 are being compared.

b. In 1990, South Asia had the greatest percentage of illiteracy for men. South Asia also had the greatest percentage of illiteracy for women.

c. In 1990, Latin America and the Caribbean had the least percentage of illiteracy for men. Latin America and the Caribbean also had the least percentage of illiteracy for women.

d. There was a higher percentage of illiterate women than illiterate men in these areas of the developing world.

e. South Asia had the greatest difference in the percentage of men illiterate and women who were illiterate.

f. Latin America and the Caribbean had the least difference in the percentage of men and women who were illiterate.

3. a. The distribution of the land surface of Earth is being shown in the circle graph.

b. Asia has the greatest amount of land.

c. North America ranks third in the amount of land area, after Asia and Africa.

d. North America has 16% of Earth's land surface.

$$\overset{\times 1\,554\,000}{\frac{16}{100} = \frac{24\,864\,000}{155\,400\,000}}$$

$\times 1\,554\,000$

There are about $24\,864\,000 \text{ km}^2$ of land in North America.

4. a. Based on the survey, the total population of junior high students is most likely to read the comics.

b. Based on the survey, the total population of junior high students is least likely to read the business section.

c. $$\begin{aligned} 38\% \text{ of } 12\,000 &= 0.38 \times 12\,000 \\ &= 4560 \end{aligned}$$

About 4560 junior high students in the city are likely to read the sports section.

d. $$\begin{aligned} 6\% \text{ of } 12\,000 &= 0.06 \times 12\,000 \\ &= 720 \end{aligned}$$

About 720 students are likely to read the editorial/letters to the editor section.

Looking Back

5. **a.** The interval with the highest number of first-borns in 1931, 1950, and 1971 was 20–24. In 1990, it was 25–29.

b. $\dfrac{\text{births to mothers under 19}}{\text{total number of births}}$

In 1931

$$\frac{14+9639}{55\ 486}=\frac{9653}{55\ 486}$$
$$\doteq 0.17$$
$$\doteq 17\%$$

In 1950

$$\frac{15+14\ 251}{92\ 018}=\frac{14\ 266}{92\ 018}$$
$$\doteq 0.16$$
$$\doteq 16\%$$

In 1971

$$\frac{292+33\ 258}{142\ 008}=\frac{33\ 550}{142\ 008}$$
$$\doteq 0.24$$
$$\doteq 24\%$$

In 1990

$$\frac{233+19\ 375}{175\ 636}=\frac{19\ 608}{175\ 636}$$
$$\doteq 0.11$$
$$\doteq 11\%$$

c. $$55\ 486-(14+9639+25\ 224+13\ 826+4802+1580+342+27)$$
$$=55\ 486-55\ 454$$
$$=32$$

The ages of the 32 mothers were not stated.

d. Answers may vary. Newfoundland may not keep data on the age of the mother at the birth of her first child.

Section 3: Activity 2

1. Answers will vary. Sample examples are given:

- amount smoked and number of sick days taken per year
- amount of time studying and examination mark
- amount of fertilizer and crop yield
- height and size of feet

2. **a.** Ms. Coulson could give the students two types of marks on their work, assignments, projects, and tests. The first mark would measure their performance, and the second mark would reflect the effort the students put into their work. If Ms. Coulson collected this data throughout the year, she would have some basis for comparison.

b. Answers will vary. Some examples are given:

- How do you measure effort (e.g., time, commitment, and attentiveness)?
- Should the students be aware they are being measured for effort? Will this influence results?
- Can Ms. Coulson be objective? Will a higher mark for effort be awarded due to a good performance mark?

3. **a.** You could set up an experiment to measure the height and the shoe size of people within various age groups, such as 5- and 6-year-olds, 15- and 16-year-olds, and 25-year-olds and over.

   b. Answers may vary. Some factors that might influence the data are age, ethnic origin, health, and weight.

4. Answers may vary. Sample questions are given.

   a. Is there a relationship between the amount of jogging (minutes per week) and longevity?

   Is there a relationship between the amount of jogging and the incidence of heart attacks?

   b. Is there a relationship between the amount of smoking (cigarettes per day) and illness (sick days per year)?

   Is there a relationship between the amount of smoking and longevity?

   c. Does increasing the amount of fertilizer used increase the amount of produce yielded?

   Does the amount of fertilizer used affect the quality of the produce yielded?

   d. Is there a relationship between the amount of time spent studying (hours per week) and the results on exams?

   Does the frequency of studying (times every week) affect the results on exams?

5. **a.** flu shot (yes/no) and number of bouts with the flu per year

   b. birth weight (in kilograms) and age at death

6. In question 5.a., the independent variable is the number of people receiving flu shots, and the dependent variable is the number of incidents of flu.

   In question 5.b., the independent variable is the weight at birth, and the dependent variable is the age at death.

7. **a.** The data should be collected from mature people—that is, people who have reached their full adult height—in order to eliminate other factors. You should have used a sample of at least 30 people and then measured their height (in centimetres) and the circumference of their head (in centimetres). Your chart should look similar to the following.

Person	Height (cm)	Head Size (cm)
Jennifer	162	56.5
Robert	180	57.4
Travis	184	57.6
Rolanda	168	56.3

b. Your sample of about 30 people can range in age. Measure their weight (in kilograms) and their wrist size (in centimetres), and record the data in a chart similar to the following.

Person	Weight (kg)	Wrist Size (cm)
Wynonna	60	15
Matt	83	17.8
Alfred	88	18.1
Roberta	72	15.5

c. Measure, to the nearest tenth of a centimetre, the diameter and the circumference of a number of circular objects (such as a tin can, a basketball, and a roll of tape). Use a string and measure carefully. Organize the data in a chart similar to the following.

Object	Diameter (cm)	Circumference (cm)
tin can	7.5	22.8
basketball	25	79
roll of tape	12.2	37.8

8. a. The two variables are mathematics marks of students in a girls-only class and the marks of students in a regular, integrated class. The relationship discussed is "Does being in a girls-only class affect the mathematics marks achieved by the girls?"

   b. Mathematics scores are improved if girls are in a class without boys.

Looking Back

9. Answers will vary. Some examples of relationships that you could investigate are as follows:

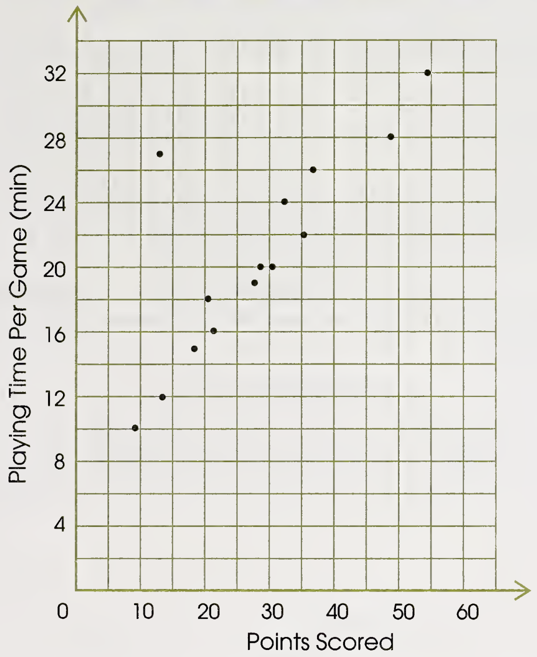
   - the price in Canadian dollars versus the price in U.S. dollars for books or magazines
   - the extension of a spring versus the mass attached
   - height versus span of hand from the tip of the outstretched thumb to the tip of the index finger
   - temperature versus time of day over a two-day period

   Your answer should include a chart similar to those in question 8 and a statement discussing the results of your experiment.

Section 3: Activity 3

1. The fuel consumption increases as the mass of the vehicle increases.

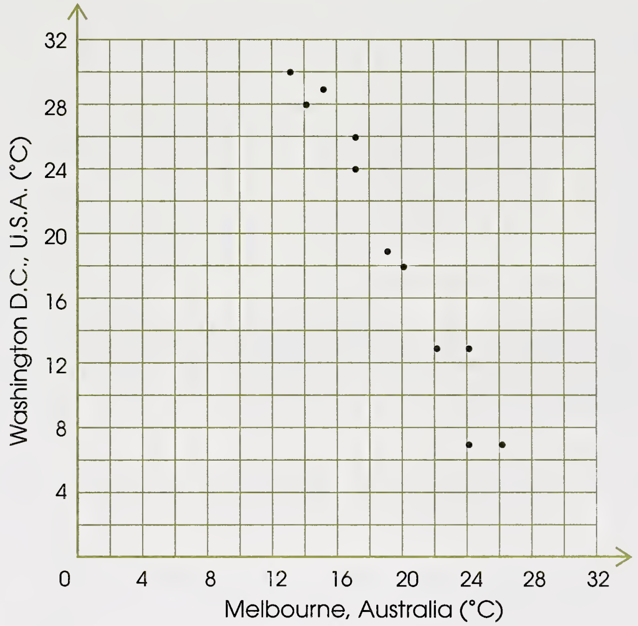
2. a. Playing Time Versus Points Scored



b. When the amount of playing time increases, the number of points scored increases.

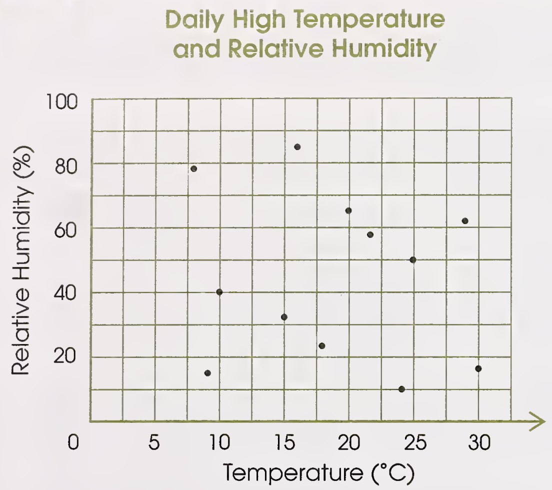
c. Yes, player #2 doesn't seem to fit the pattern. Perhaps the player plays defence and is noted for defensive ability rather than scoring prowess.

3. Average Monthly Temperature of Two Cities



As the temperature rises in Washington D.C., the temperature in Melbourne decreases and vice versa. This relationship occurs because Washington D.C.'s and Melbourne's seasons are opposite one another. For example, winter in Washington D.C. coincides with summer in Melbourne.

4.



There doesn't seem to be any trend or noticeable relationship between temperature and relative humidity.

5. **a.** The points plotted will rise to the right. You would expect as one spouse ages, the other would be relatively close in age.

   b. The points plotted will rise to the right. Income increases with years of post-secondary education.

   c. The points plotted will show no particular relationship.

   d. The points plotted will rise to the right. The higher the temperature, the more ice cream sold.

   e. The points plotted will fall to the right. The faster you go the shorter the time it will take to arrive.

Looking Back

6. Answers will vary.

Section 3: Activity 4

1. Yes, the line is a good approximation.

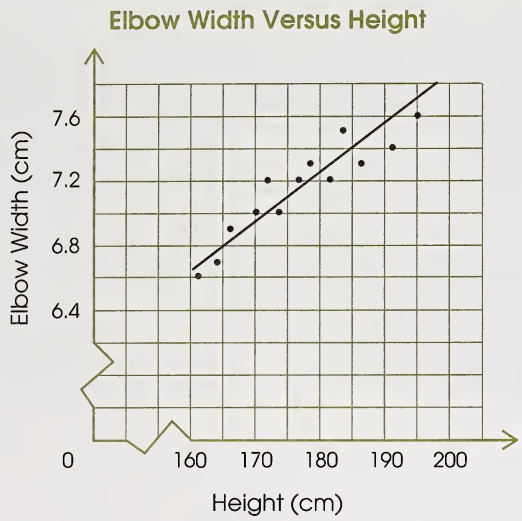
2. The line suggests that vehicle fuel consumption increases as mass increases.

3. **a.** The fuel consumption of a vehicle with a mass of 1900 kg is about 13 L/100 km.

   b. According to the scatter plot, the mass of a vehicle with a fuel consumption of 3.5 L/100 km would be about 500 kg.

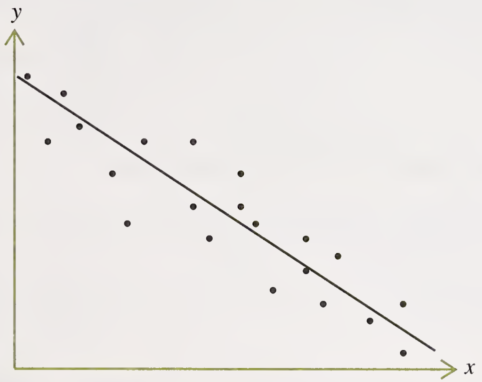
   c. The line of best fit represents the average fuel consumption for a vehicle with a given mass. The vehicles represented by the points not on the line of best fit are more or less fuel efficient than the average vehicle.

4. Your line of best fit should look somewhat like the following.

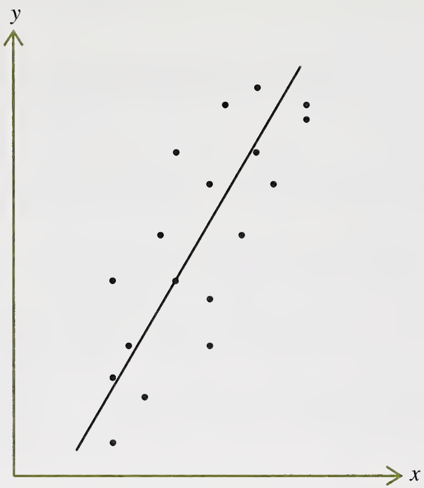


5. These lines of best fit are approximations. Your lines may not be exactly the same, but they should be close to the ones given.

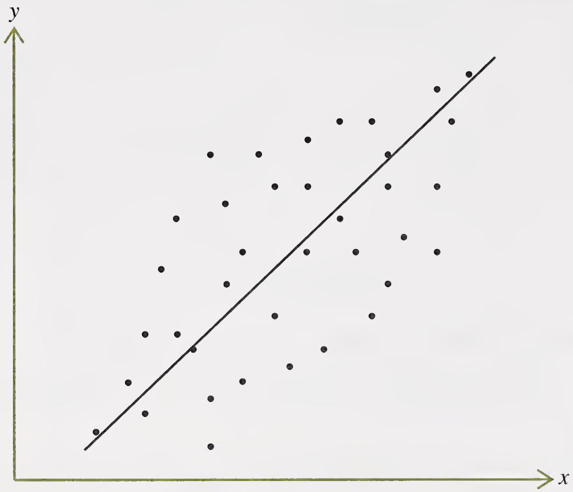
a.



b.



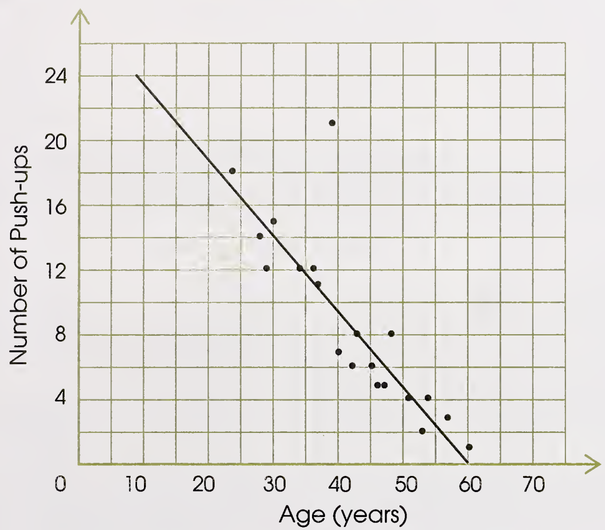
c.



6. You should have found it easiest to locate the line of best fit on the scatter plot in question 5.a., since all the points are closest to being in a line.

7. a. Your scatter plot and line of best fit should look as follows.

Number of Push-ups by the Female Staff at Eastside High School



b. As each female staff member gets older, the number of push-ups they can do decreases.

c. The one exception could be a physical education teacher or someone who works out on a regular basis.

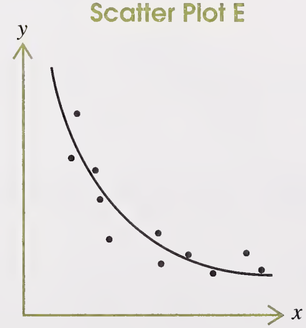
d. You should have ignored the one point that was an exception when drawing your line of best fit.

e. According to the graph, staff that are 60 years old and over cannot do any push-ups. This may not be true for society as a whole.

8. a. Scatter plots A, B, C, and E show some kind of trend.

b. You would draw a smooth curve through the points in scatter plot E.

Scatter Plot E



c.

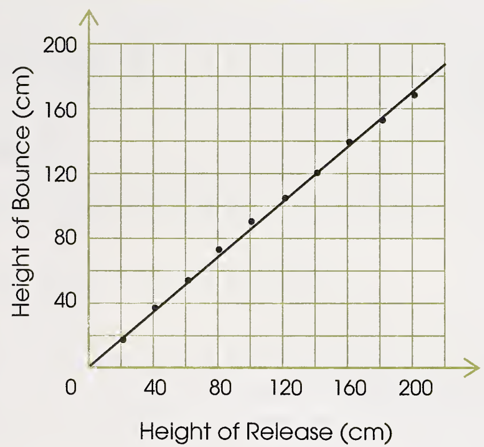
The data has no apparent trend. There does not appear to be any relationship between the variables.

9. **a.** Answers will vary. A sample chart and scatter plot are given.

Height of a Bouncing Ball

Height of Release (cm)	Height of Bounce (cm)	Height of Release (cm)	Height of Bounce (cm)
20	17	120	103
40	37	140	120
60	52	160	138
80	72	180	152
100	90	200	168

Height of a Bouncing Ball



b. The higher the ball is when dropped, the higher it bounces.

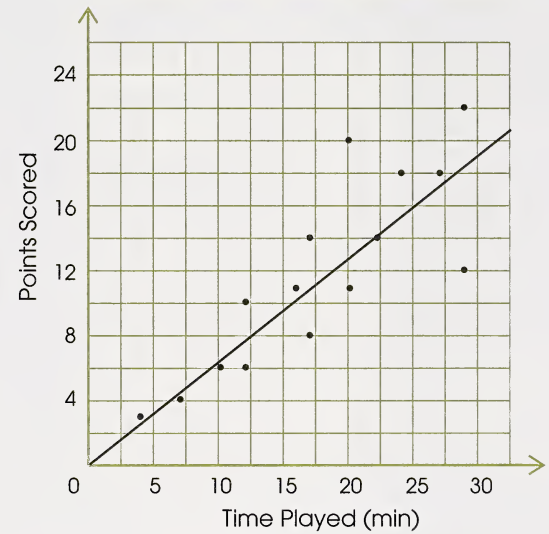
c. Some points may be further from the line of best fit due to measuring error.

d. Answers will vary depending on your experimental results. In this case, a ball dropped from 100 cm will bounce about 85 cm.

e. Yes, the type of ball would make a difference.

f. Answers will vary. You should have found that a hard ball, like a superball, will bounce the highest.

10. **a.** Points Scored Versus Time Played



b. The number of points scored increases as the playing time increases.

c. Is Ryning contributing in other ways to the team (rebounds, assists, blocked shots, and so on) or should her playing time be decreased? Perhaps House should be getting more playing time based on her scoring average.

d. A player should yield about 16 points.

e. The player might be able to score about 19 points.

Looking Back

11. All the points should be as close as possible to the line of best fit in order to make good predictions.

Credits

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